EXPERIMENTAL STUDY OF CORRELATOR FILTERS NOT UTILIZING A DELAY

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THESIS

Experimental Study of
Correlator Filters Not
Utilizing a Delay

by

Tito Manlio Rincon

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To my wife, Omaira and my children, for their confidence and assurance.

I. INTRODUCTION

The detection of a signal in the presence of noise requires a process which becomes more complex when little is known of the noise characteristics and of the signal. To this end, correlation analysis is one of the most valuable techniques used to filter noise from very low level signals. In fact, the problem of getting the signal out of noise is the major function of analyzing data in many areas today.

When properly used and understood, correlation provides the engineer with a technique as powerful as Fourier Analysis or any of the other classical techniques. Essentially, correlation analysis and Fourier transform can be thought of as duals of each other since in general the transform of any extended signal in time is a narrow frequency signal and viceversa, and the autocorrelation of a single frequency sine wave extends over the entire time delay axis. Familiarization then with this technique and the ability to shift from the time to frequency to delay axes in order to extract more and more useful information from a multitude of signals is of the most importance for communications and related fields.

The mathematical background of the correlation functions, autocorrelation and crosscorrelation, is presented first,[Ref. 1,2,3,5] together with their properties and Fourier transform correspondence. Then, the method of obtaining the correlation functions are briefly explained. In particular, the practical application of the correlation method without a "pure" time delay is presented. This method is based on the application of the concept of orthogonal filters which are Laguerre Functions type filters.

A practical application of this method was carried out in the laboratory to verify the analysis. The analysis was done by means of the DSL subroutine which simulated the

correlator. The experimental results of the autocorrelation of a sine wave of 155 Hz were as predicted by the simulation.

II. CORRELATION ANALYSIS.

Correlation analysis is a technique used to determine the spectral characteristics of a signal or the similarity of two different signals.

A good method of measuring the similarity between two signals is to multiply them together, ordinate by ordinate and to add the products over the duration of the waveforms. The result is a single number which represents the similarity between the two wave forms.[Ref. 3]

The two correlation functions, autocorrelation and crosscorrelation, can be explained as follows [Ref. 1,2]:

Suppose that a stationary, physical process is producing the time functions $x_1(t)$, $x_2(t)$, ... $x_n(t)$ simultaneously. (A process is said to be stationary if its amplitude statistics do not change with time). It is also assumed that:

(1) The time functions being generated are not zero. Or they have zero mean if the time functions correspond to a stochastic process

(2) They do not have a DC component.

(3) They can be simple or complex periodic waves.

(4) They may vary in a random fashion.

Under these conditions, the two correlation functions are defined as:

A. AUTOCORRELATION FUNCTION. [Ref. 1,2]

The time average autocorrelation function for deterministic or known signals is defined for two different classes of signals:

a) signals of finite average power.

b) signals of finite energy or pulse signals

For signals of finite average power the definition is:

$$R_{xx}(u) = \lim_{T\to\infty} 1/2T \int_{-T}^{T} x_1(t)\,x_1(t+u)\,dt \qquad (2.01)$$

where "u" represents a time-shift.

For signals of finite energy the definition is:

$$R_{xx}(u) = \int_{-\infty}^{\infty} x_1(t)\,x_1(t+u)\,dt \qquad (2.02)$$

The autocorrelation of functions generated by a stochastic process is defined as:

$$R(u) = E[\; x_1(t)\,x_1(t+u) \;] \qquad (2.03)$$

and if the process is ergodic, the autocorrelation function is defined as:

$$R_{xx}(u) = R(u) \qquad (2.04)$$

In general, the autocorrelation function of a waveform $x_1(t)$ is a graph of the similarity between the waveform $x_1(t)$ and a delayed replica of itself, $x_1(t+u)$ as a function of the time shift "u".

Based on the mathematical definition of autocorrelation, Figure 1 shows an elementary device designed to obtain the autocorrelation function. Here, the signal is multiplied by its delayed replica and the result is averaged over a sufficiently long time. The final result is the autocorrelation function if u has also been varied during the process. This is essentially a time-delayed autocorrelator.

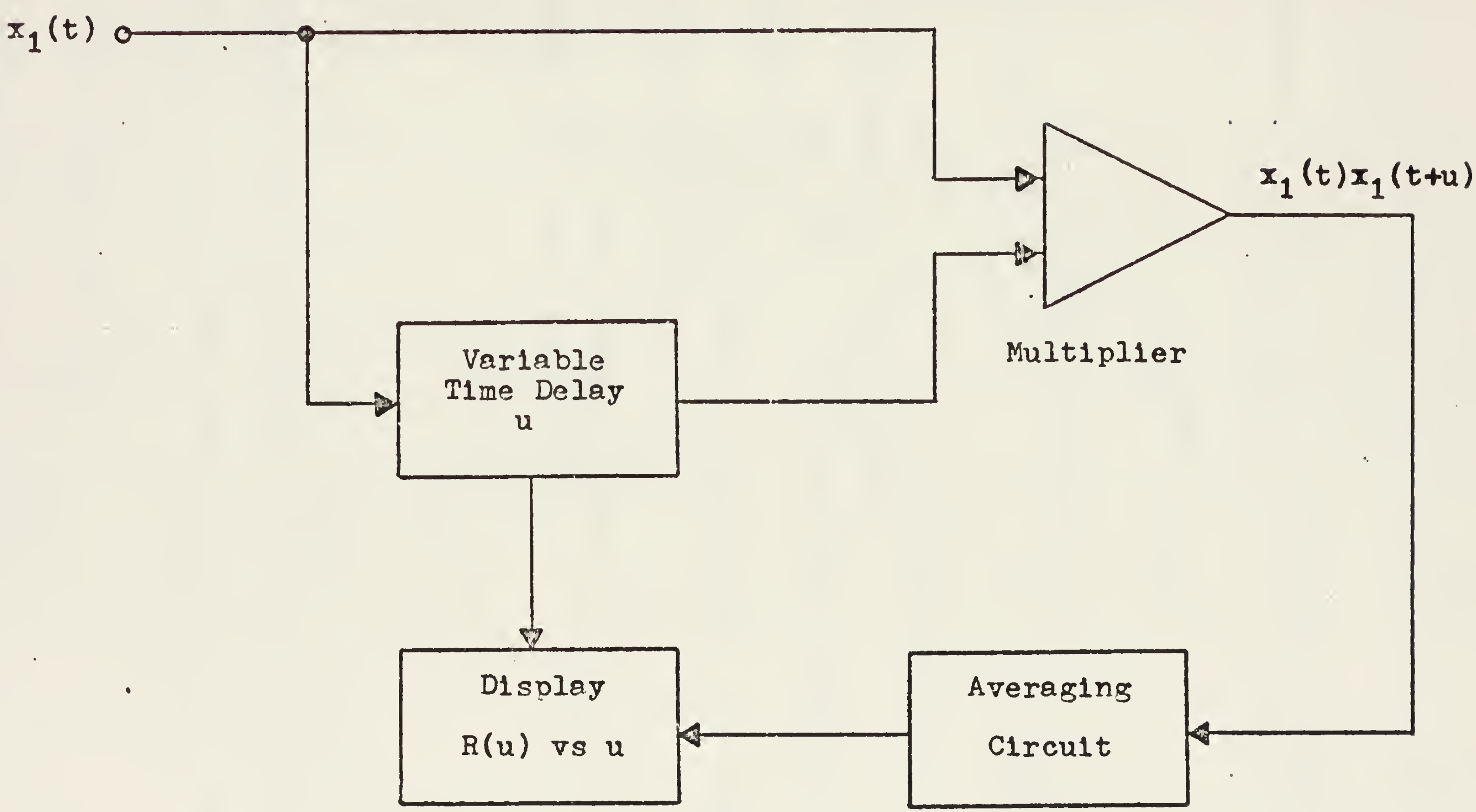


Figure 1: Autocorrelator Block Diagram.

1. <u>Properties of the Autocorrelation Function.</u>

The final output of figure 1, that is, the autocorrelation function, will always have the following characteristics for any $x(t)$:

(1) When $u = 0$ the product is maximum. This is represented mathematically by Schwartz Inequality:

$$|R_{xx}(u)| \leq R_{xx}(0) \tag{2.05}$$

(2) The value of the output at $u = 0$ represents the total power of the signal. If $x_1(t)$ is a voltage, then for $u = 0$, $R_{xx}(0) = \overline{x_1(\tau)}^2$ is the average power of the signal (as measured on a 1 ohm resistor).

(3) The autocorrelation function is also a function of u.

(4) The shape of the function is characteristic of the original signal $x_1(t)$.

(5) The autocorrelation function is an even function of u. Mathematically:

$$R_{xx}(u) = R_{xx}(-u) \tag{2.06}$$

(6) If $x_1(t)$ contains periodic frequency components, then $R_{xx}(u)$ will contain each of these frequency components. This means that for certain time functions, if $x_1(t)$ is given then $R_{xx}(u)$ is known which is useful in detecting signals by radar.

(7) For a real process of this kind, the autocorrelation function approaches zero as $u \to \infty$. In other words, the signal loses coherence as the delay u is

increased. So the coherence time of the original signal is measured by that value of u that produces a significant reduction in the value of the autocorrelation function. The coherence time of any process producing very wideband, uniform (white) noise, is practically zero or very short because its instantaneous value is nearly independent of the value at any other time.

(8) The autocorrelation function and the power spectral density are Fourier transform pairs. So they contain the same information, however, the autocorrelation function contains this information in the form of a function of time rather than frequency.

Mathematically:

$$\phi(w) = \int_{-\infty}^{\infty} R_{xx}(u)\, e^{-jwu}\, du \tag{2.07}$$

$$R_{xx}(u) = 1/2\pi \int_{-\infty}^{\infty} \phi(w)\, e^{jwu}\, dw \tag{2.08}$$

The power spectral density of finite average power signals is:

$$\phi(w) = \lim_{T\to\infty} 1/2T \; X_{1T}(w) X_{1T}^{*}(w) \tag{2.09}$$

$$= \lim_{T\to\infty} 1/2T \; |X_{1T}(w)|^{2} \tag{2.10}$$

where * means the conjugate value of the quantity it refers to. The signal has been observed through a window of observation of duration T.

The power spectral density for signals of finite energy is:

$$\phi(w) = X_{1}(w) X_{1}^{*}(w) = |X_{1}(w)|^{2} \tag{2.11}$$

This is also true for stochastic processes in which case:

$$R_{xx}(u) \iff \phi(w) \tag{2.12}$$

and

$$\phi(w) = \lim_{T} 1/2T \; E\{|X_{1T}(w)|^2\} \tag{2.13}$$

If the Fourier components of the waveform are squared in amplitude, set into phase at the origin of a new time scale, and added together, the result is a visual picture of the autocorrelation function.

(9) The process of autocorrelation of a waveform is equivalent to passage of the waveform through its matched filter.

Figure 2 shows the autocorrelation function of some typical waveforms.

B. CROSSCORRELATION FUNCTION. [Ref. 1,2,3]

Autocorrelation can be easily visualized because it can be related to the power density spectrum of Fourier Analysis. But crosscorrelation does not have a similar analogy.

Crosscorrelation is concerned with the relationship between two different signals generated by some common process. In general, the crosscorrelation function of two waveforms, $x_1(t)$ and $x_2(t)$ is a graph of the similarity between $x_1(t)$ and the delayed $x_2(t)$, $x_2(t-u)$ as a function of the delay "u" between them.

Mathematically, crosscorrelation is expressed as:

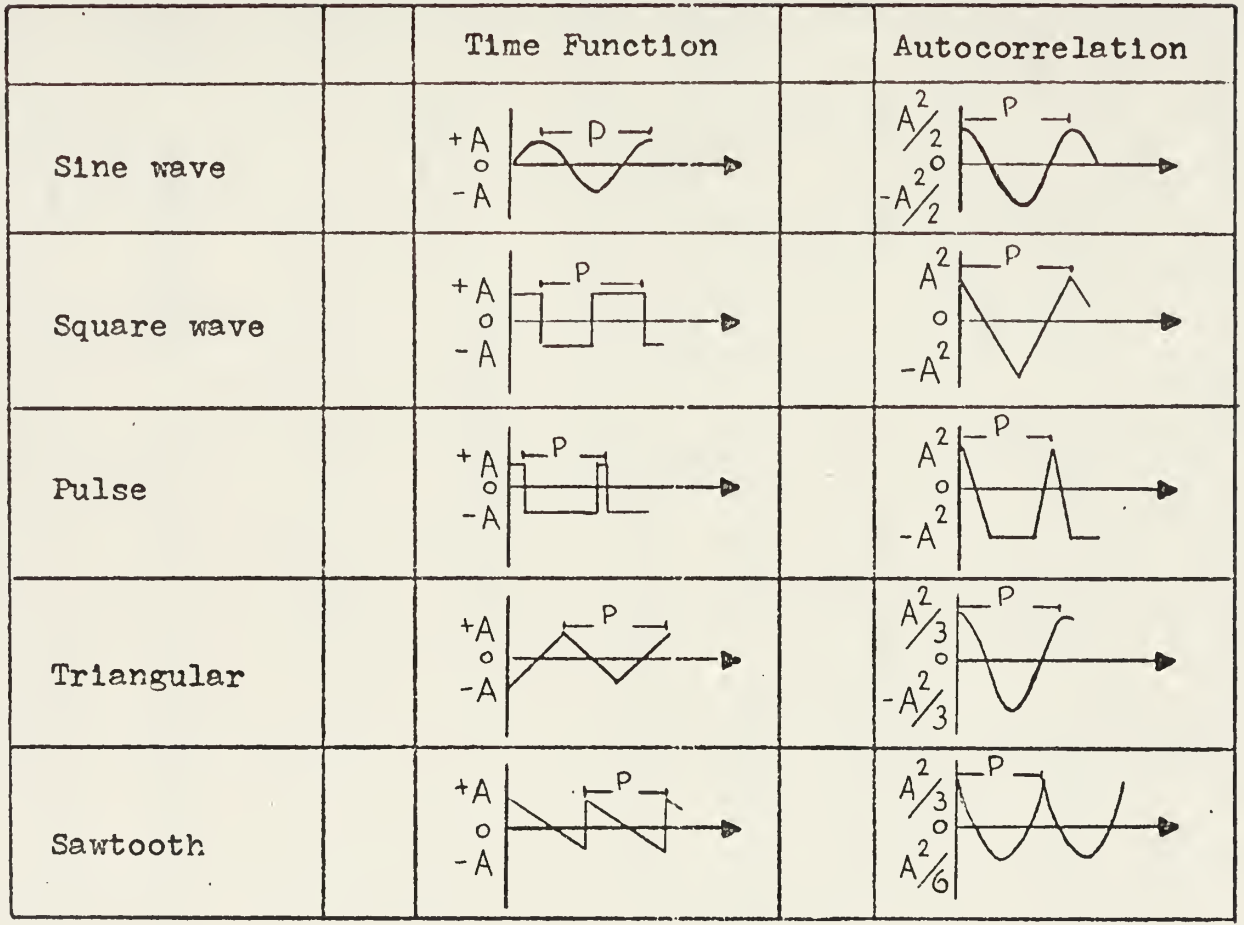


Figure 2: Autocorrelation Functions of some typical Waveforms.

$$R_{x_1 x_2}(u) = \lim_{T} 1/2T \int_{-T}^{T} x_1(t)\, x_2(t+u)\, du \qquad (2.14)$$

Figure 3 shows a device, similar to that shown in Figure 1, designed to obtain the crosscorrelation function from its mathematical definition. In this case, one of the two signals is delayed, multiplied by the other signal and the result averaged over a long enough period of time. The result is the crosscorrelation function provided that u has also been varied over the process. This is also a time-delayed crosscorrelator.

1. Properties of the Crosscorrelation Function.

The autocorrelation and the crosscorrelation functions have different properties. The properties of the crosscorrelation function are:

(1) The crosscorrelation function is not an even function of u. In other words:

$$R_{xy}(u) \neq R_{xy}(-u) \qquad (2.15)$$

However

$$R_{xy}(-u) = R_{yx}(u) \qquad (2.16)$$

and this relationship is useful for obtaining $R_{xy}(u)$ for negative delays.

(2) The coherence of a signal generated by a physical process approaches zero very fast as u approaches infinity due to the presence of noise and the uncertainty principle. If, added to this, $x_1(t)$ and $x_2(t)$ come from two different, unrelated processes, then:

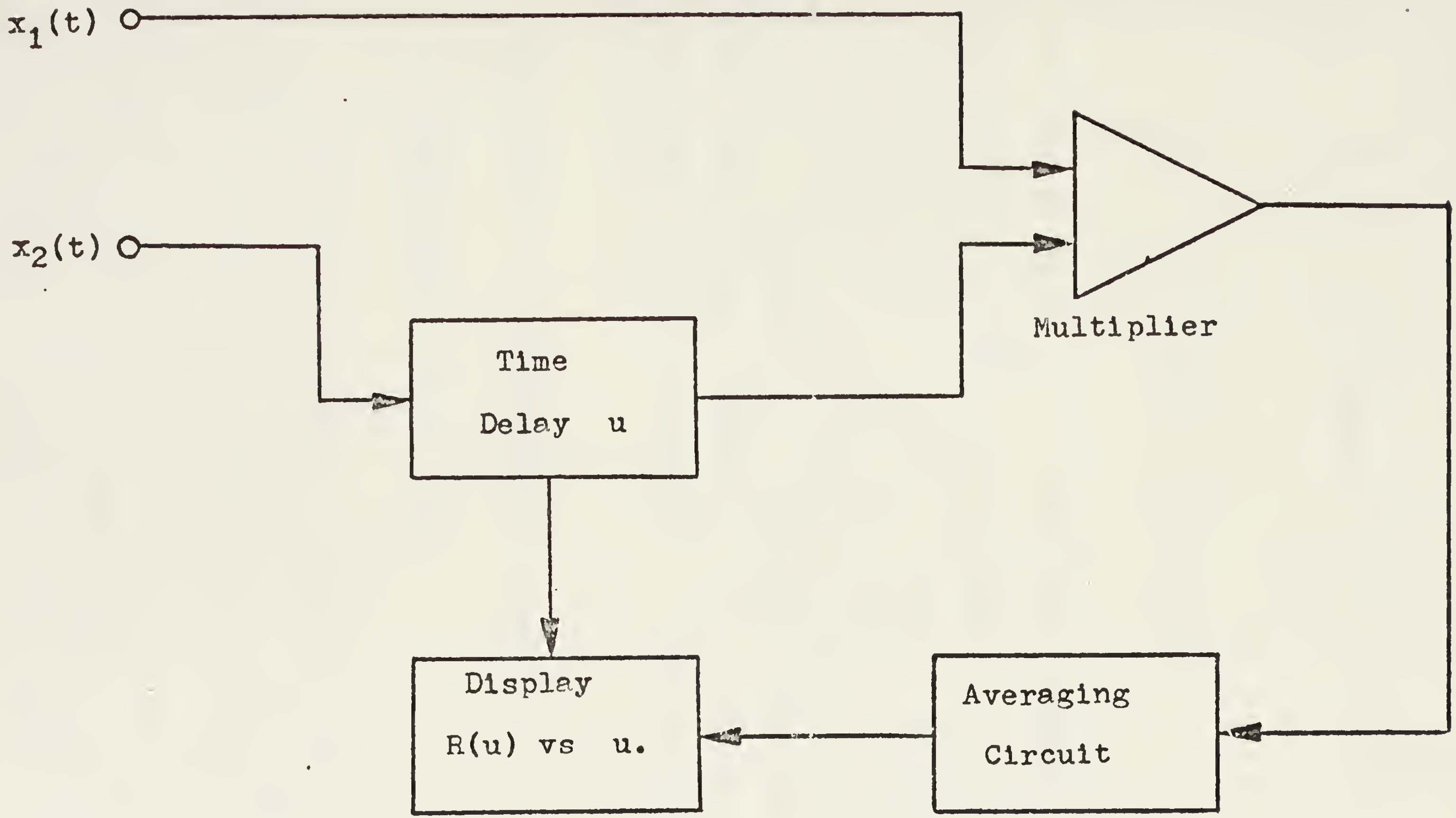


Figure 3: Crosscorrelator Block Diagram.

$$R_{xy}(u) = 0 \qquad (2.17)$$

(3) In the frequency domain there exists a Fourier transform pair:

$$R_{xy}(u) \Longleftrightarrow W_{12}(w) \qquad (2.18)$$

but in this case, $W_{12}(w)$ is sometimes called the spectrum of crosscorrelation of the time functions $x_1(t)$ and $x_2(t)$

Crosscorrelation analysis provides a powerful analytical tool. The ability to measure the degree to which the signals that arise from a common physical phenomenon resemble each other as a function of the delay time between them, can provide a much deeper insight into the phenomenon being studied than a separate analysis of the properties of either signal alone. [Ref. 1]

C. APPLICATIONS OF CORRELATION ANALYSIS.

1. Detection. [Ref. 1,7]

Correlation analysis plays its most important role when noise is to be filtered from very low level signals. In this case, noise is any undesired disturbances that mask the signal transmitted, and in practice, it is a signal with random amplitude variations. In the case of wide band noise, the instantaneous value of the signal is nearly independent of the value at any other instant which means that the coherence time of the process is very short.

Signal detection, such as radar detection, is basically the solution of a very important problem common to all echo-ranging systems: a signal of known waveform is transmitted into a medium and is received, unchanged in form but inmersed in noise.

When two functions consisting of a signal, $S(t)$, and noise, $N(t)$, such as:

$$x_1(t) = S_1(t) + N_1(t) \tag{2.19}$$

$$x_2(t) = S_2(t) + N_2(t) \tag{2.20}$$

are crosscorrelated, the result is:

$$R_{xx}(u) = \lim_{T\to\infty} 1/2T \int_{-T}^{T} [S_1(t)+N_1(t)][S_2(t+u)+N_2(t+u)]dt \tag{2.21}$$

$$= R_{S_1 S_2}(u) + R_{S_1 N_2}(u) + R_{N_1 S_2}(u) + R_{N_1 N_2}(u) \tag{2.22}$$

which will approach zero as u approaches infinity.

If $x_1(t) = x_2(t)$, then the crosscorrelation is:

$$R_{xx}(u) = R_{SS}(u) + R_{SN}(u) + R_{NS}(u) + R_{NN}(u) \tag{2.23}$$

but in this case,

$$R_{SN}(u) = R_{NS}(u) = 0 \tag{2.24}$$

because the signal S(t) and the noise N(t) are uncorrelated.

$R_{SS}(u)$ is the autocorrelation function of the original signal and it is a function of u other than zero even for large values of u. $R_{NN}(u)$ is the autocorrelation function of noise. If the transmitted signal is periodic then $R_{NN}(u)$ becomes very small for large values of u while $R_{SS}(u)$ does not. so $R_{XX}(u)$ is almost equal to $R_{SS}(u)$ and this result can be used for signal detection.

In the case of radar detection, the transmitted and the received signals are crosscorrelated. Let

$$x_1(t) = S_1(t) + N(t) \tag{2.25}$$

be the received signal and

$$x_2(t) = S_2(t) \tag{2.26}$$

be the transmitted signal. The frequency of the transmitted signal is known so that an internal generator in the receiver can be used to generate $x_2(t)$ as a second input to the correlator. When these two signals are crosscorrelated, the result is:

$$R_{12}(u) = R_{S_1S_2}(u) + R_{NS_2}(u) \tag{2.27}$$

since $N(t)$ and $S_2(t)$ are not correlated, then

$$R_{NS_2}(u) = 0 \tag{2.28}$$

eventually. So

$$R_{12}(u) = R_{S_1S_2}(u) \tag{2.29}$$

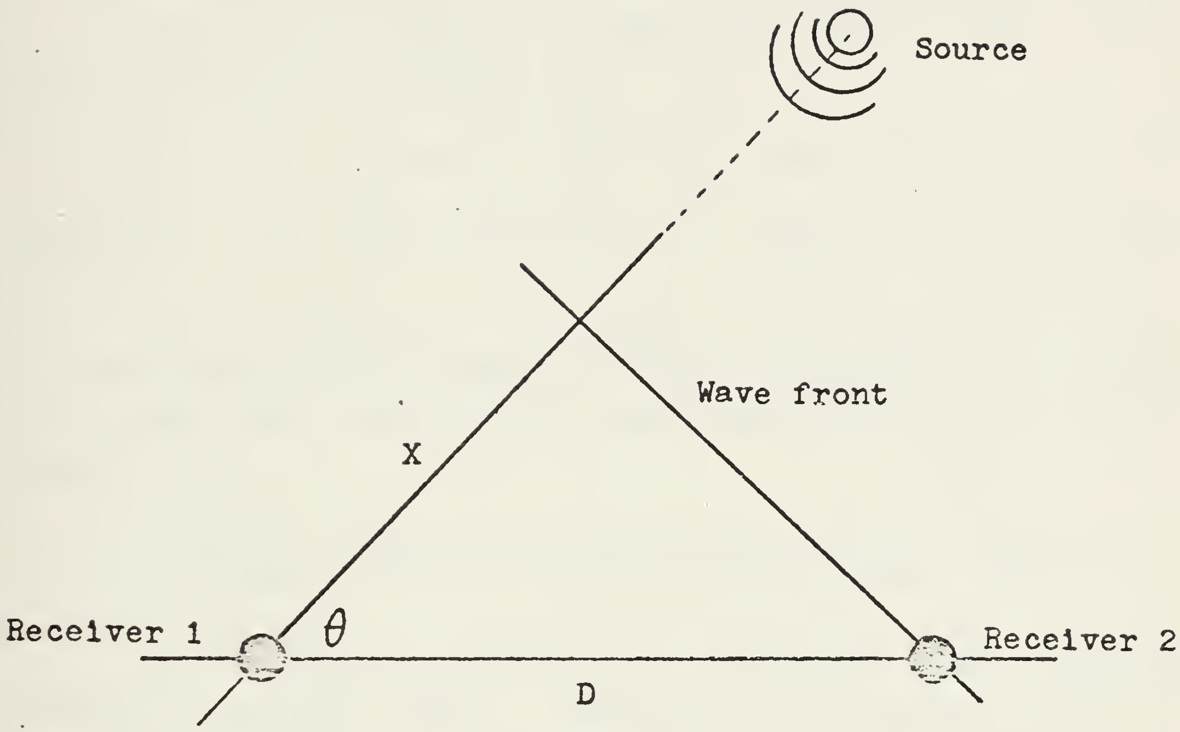
If $x_1(t)$ is only noise $[S_1(t)=0]$, then

$$R_{S_1S_2} = 0 \tag{2.30}$$

so $R_{12}(u) = 0$. This means that if the crosscorrelation between transmitted and received signal is not identically zero, then a useful signal exists in the received transmission.

2. <u>Direction Finding.</u> <u>[Ref. 3]</u>

Direction finding is another application of crosscorrelation. In this case, the direction of a source of arbitrary waveforms (acoustic, electromagnetic, seismic, etc), can be determined by crosscorrelating the responses at two receivers that may be located far from the source. The procedure is represented in figure 4. The waveforms can be completely arbitray, and can even be noise. In the figure, the wave front gets to receiver 2 first and it will get to receiver 1 t seconds later where $t = (D/c)\cos\theta$ and c is the velocity of propagation. The crosscorrelation has its maximum at a delay time equal to the difference in



$X = D\cos\theta$

$t = (D/c)\cos\theta$: Time difference between receptions at receivers.

c = Velocity of propagation.

Figure 4: Direction Finding.

propagation time from the source to each of the two receivers. Since D is a constant, then the direction to the source, which is the angle θ measured between the propagation direction and the base line between receivers, is a function of the delay t (time difference). So, if a source is at θ_1 and another is at θ_2 then the crosscorrelation of the signals at the receivers from both sources will peak at two different times.

This technique can be used to find the direction of any disturbance which radiates a plane wave such as cosmic noise sources, earthquakes, submarines, and radar or sonar targets.

3. Testing Control System Response On-Line. [Ref. 3]

It is often necessary to determine the transfer function or the impulse response of a control system or plant which may be in continuous use. In this case, the adjustment should be done without taking the control system out of work. Crosscorrelation is idealy used for these cases, and the adjustment can be performed on-line.

Figure 5 represents a typical control system. Low-level wideband noise is introduced into the servo loop and it is crosscorrelated with the output. The control system acts as a filter and the crosscorrelation represents the impulse response of that system. Figure 6 represents a typical control system response.

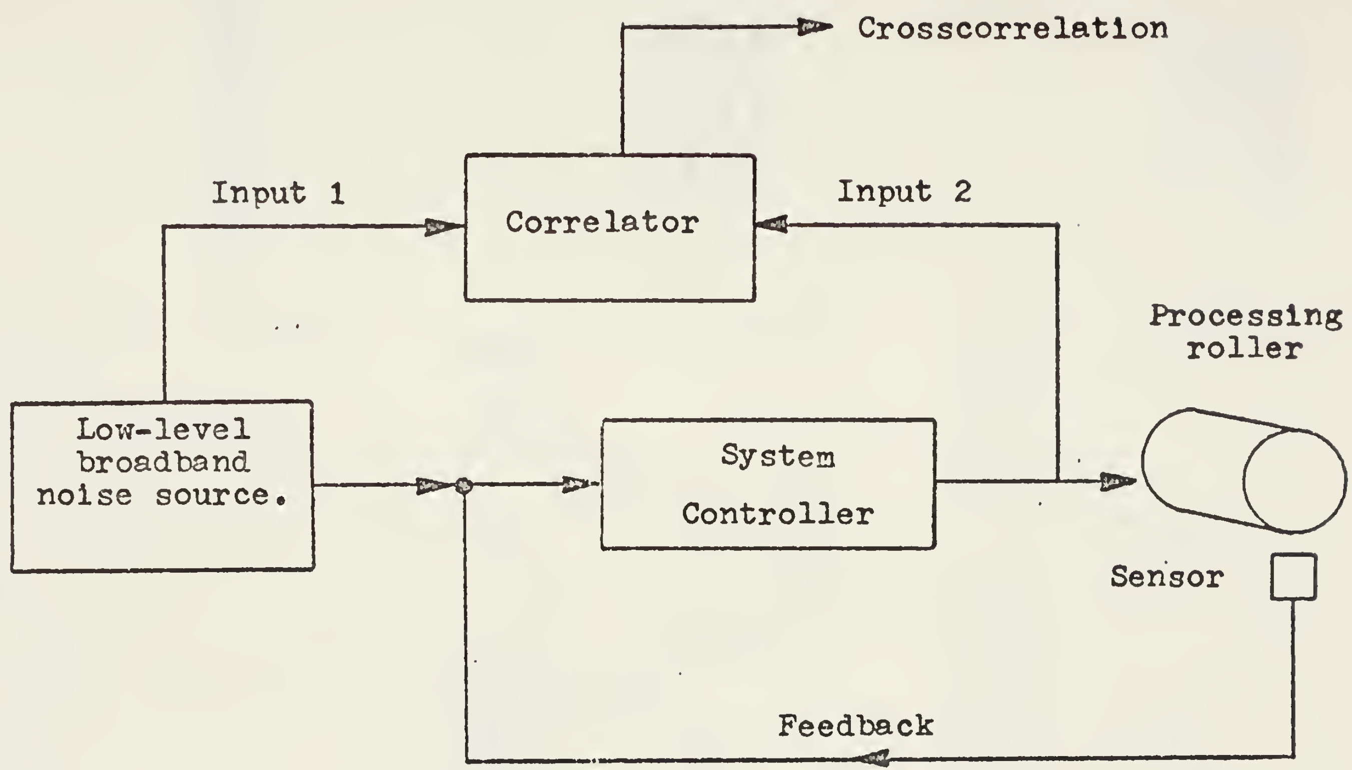


Figure 5: Block Diagram of a Control System.

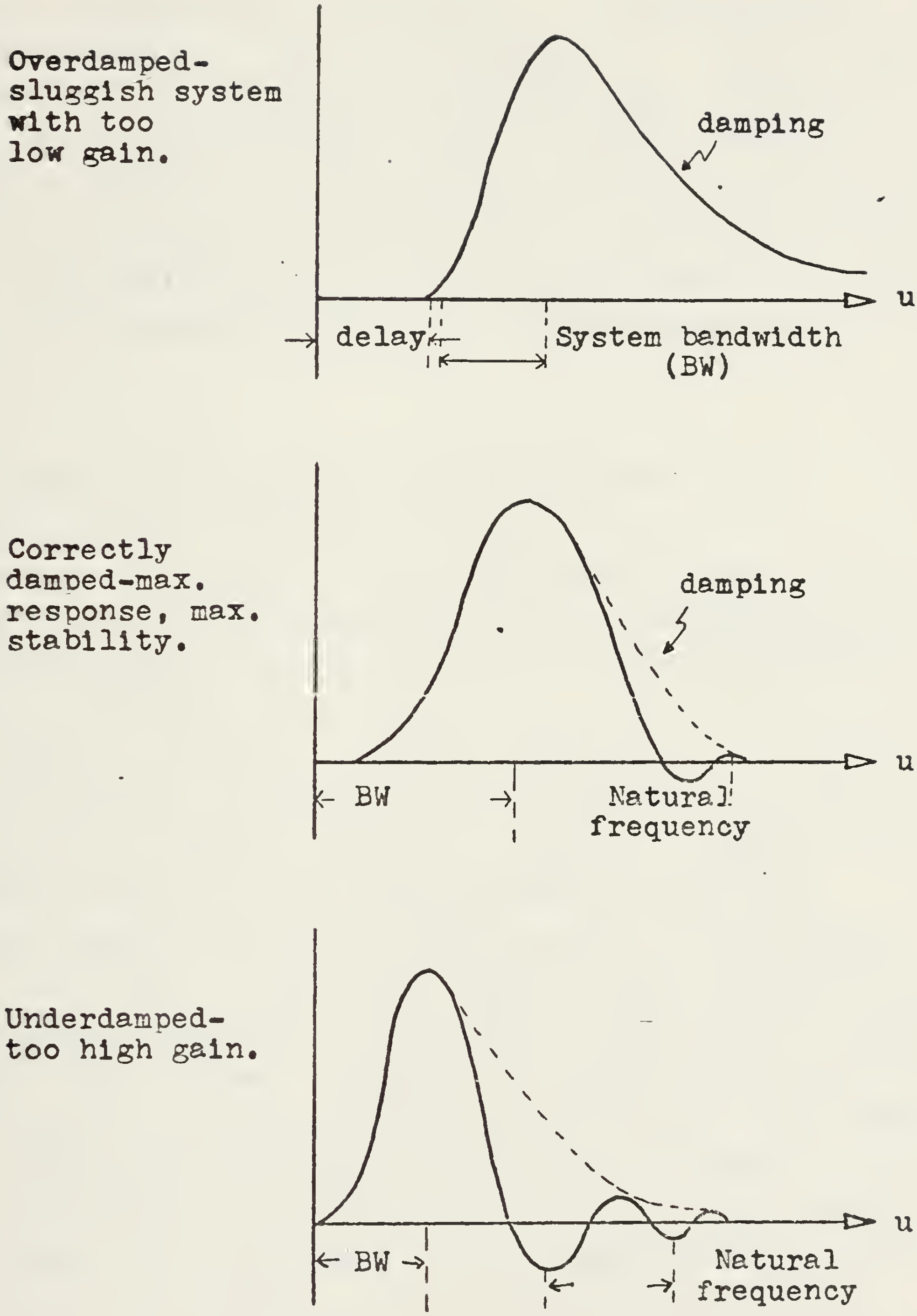


Figure 6: Typical Control System response.

III. GENERATION OF THE CORRELATION FUNCTIONS.

The correlation functions, autocorrelation and crosscorrelation, can be generated by three different methods. The first two methods (time-domain definition and indirect frequency-domain) will be briefly explained in this section. The third method (method without pure time delay) will be explained in the follwing sections. This last method can generate the correlation functions in a fairly easy and inexpensive way and it is the main concern of this thesis.

A. METHOD I: TIME-DOMAIN DEFINITION. [Ref. 1]

This is an expensive but widely used method. Here, the correlation functions are generated by a simple application of the mathematical definition of autocorrelation and crosscorrelation.

Figure 1 shows a simple device to obtain the autocorrelation function and it consists in a step by step solution of equation (2.01). Figure 3 shows the same application but for the crosscorrelation function.

The method consists in delaying one of the signals "u" units of time, then both signals under consideration are multiplied together and the product is fed into a low-pass filter. The filter output is one point of the correlation function. The complete correlation function is generated when the delay between the two signals is varied. Note that if the two functions being correlated are equal except for the delay "u", the result is the autocorrelation function. Otherwise the result is the crosscorrelation function.

There are some disadvantages to this method, mainly:

(1) If the two signals are fluctuating, and if the delay "u" is too long, a distortion is introduced in the system.

(2) Since this method is a discrete process, the total

number of points which generate the complete correlation function depends upon the number of individual delay devices used.

B. METHOD II: INDIRECT OR FREQUENCY-DOMAIN METHOD.

One of the properties of the correlation functions is the fact that they are related to the power spectrum through the Fourier Transforms, the autocorrelation being the inverse transform of the auto-spectrum and the cross-correlation the inverse of the cross-spectrum. Thus, the indirect method consists in transforming to and from the frequency domain so that the correlation in the time domain is equivalent to a complex conjugate multiplication of the signals spectra in the frequency domain.

This method is expensive and its application is based on the Fast Fourier Transform algorithm which is a descrete (digital) process and is mainly used for special purpose machines. [Ref. 1,8]

C. METHOD III: METHOD WITHOUT PURE TIME DELAY.

The third method used to generate the correlation functions is based on the fact that either autocorrelation or crosscorrelation can be represented as a series expansion of orthogonal terms. The problem is now how to obtain the coefficients of the series expansion and how to use them in order to get practical results in a fairly inexpensive and simple way. This method will be fully discussed in the following sections.

IV. CORRELATION ANALYSIS WITHOUT TIME DELAY.

A. THEORY. [Ref. 2]

The autocorrelation function can be expanded in a series of orthogonal functions as follows:

$$R(u) = \sum_{n=0}^{\infty} a_n \theta_n (u)[p(u)]^g \qquad 0 \le u < \infty \qquad (4.01)$$

where

a_n : coefficients of the series expansion.

$p(u)$: a weight function to be defined later.

$\theta_n(u)$: polynomials that form an orthonormal set with respect to $p(u)$ in the range $0 \le u < \infty$.

The coefficients a_n are given by:

$$a_n = \int_0^{\infty} R(u)\theta_n(u)[p(u)]^{1-g} du \qquad (4.02)$$

1. Orthogonal Filters. [Ref. 1,2]

Figure 7 shows a system that can be used to generate the coefficients a_n.

If it is supposed that the linear network has an impulse response $h_n(t)$, then the filter output is:

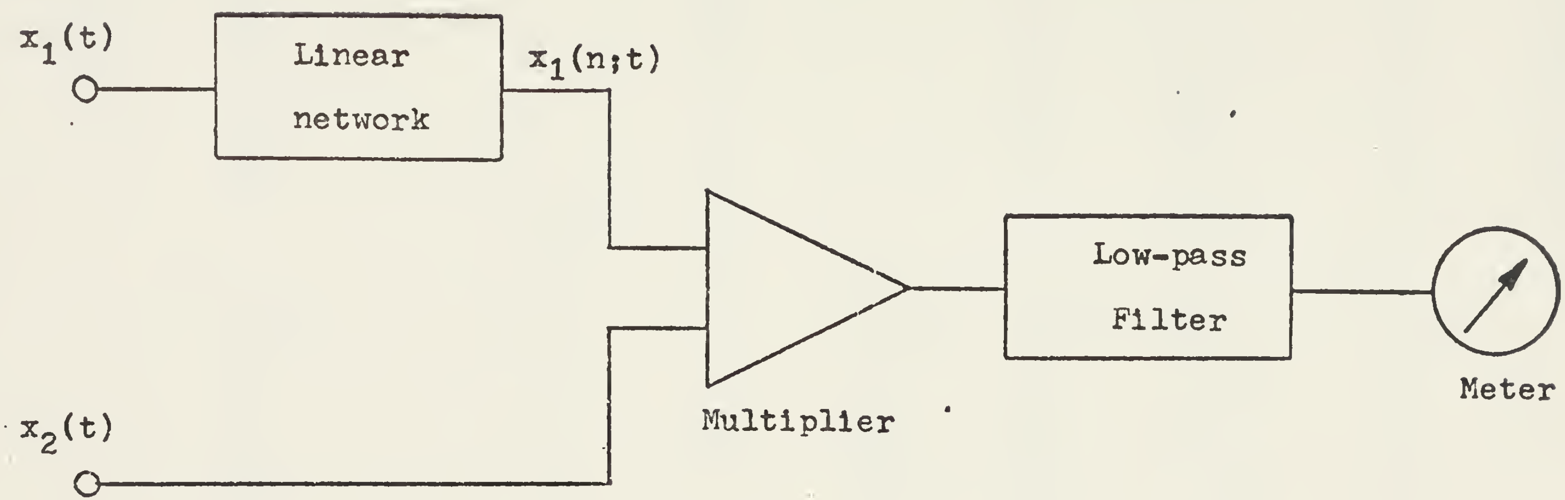


Figure 7: Basic System to obtain the Coefficients a_n.

$$x_1(n;t) = \int_0^{\infty} x_1(t-v)\,h_n(v)\,dv \tag{4.03}$$

and the multiplier output is:

$$x_1(n;t)\,x_2(t) = \int_0^{\infty} x_1(t-v)\,x_2(t)\,h_n(v)\,dv \tag{4.04}$$

The output of the low-pass filter is the mean or average value of equation (4.04) So:

$$\overline{x_1(n;t)\,x_2(t)} = \int_0^{\infty} \overline{x_1(t-v)\,x_2(t)}\,h_n(v)\,dv \tag{4.05}$$

$$= \int_0^{\infty} R(v)\,h_n(v)\,dv \tag{4.06}$$

By comparing equations (4.02) and (4.06) namely:

$$a_n = \int_0^{\infty} R(u)\,\theta_n(u)\,[p(u)]^{1-g}\,du \tag{4.02}$$

and

$$\overline{x_1(n;t)\,x_2(t)} = \int_0^{\infty} R(v)\,h_n(v)\,dv \tag{4.06}$$

it can be seen that

$$a_n = \overline{x_1(n;t)\,x_2(t)} \qquad (4.07)$$

if $h_n(t)$ is chosen to be

$$h_n(t) = \theta_n(t)[p(t)]^{1-g} \qquad (4.08)$$

Equation (4.08) is a definition. Any filter whose impulse response satisfies equation (4.08) is defined as an orthogonal filter.

The above development shows that at least mathematically, the coefficients a_n of the series expansion can be obtained. Actually, it will be shown next that the orthogonal filters can be realized and the coefficients a_n can be obtained from a practical system which uses a finite number of these filters.

2. <u>Synthesis Problem.</u>

Equation (4.01)

$$R(u) = \sum_{n=0}^{\infty} a_n \theta_n(u)[p(u)]^{g} \qquad 0 \leq u < \qquad (4.01)$$

can be written as:

$$R(u) = [p(u)]^{2g-1} \sum_{n=0}^{\infty} a_n \theta_n(u)[p(u)]^{1-g} \qquad (4.09)$$

or

$$R(t) = [p(t)]^{2g-1} \sum_{n=0}^{\infty} a_n h_n(t) \qquad 0 \le t < \infty \qquad (4.10)$$

where

$$h_n(t) = \theta_n(t)[p(t)]^{1-g} \qquad (4.08)$$

for an orthogonal filter.

In a practical system, only a finite number of filters is used. If N filters are used then equation (4.09) can be approximated as:

$$R_N(t) = [p(t)]^{2g-1} \sum_{n=0}^{N} a_n h_n(t) \qquad (4.11)$$

If g=1/2, which is called the "symmetry" case, then

$$R_N(t) = \sum_{n=0}^{N} a_n h_n(t) \qquad (4.12)$$

and if g = 0 then:

$$R_N(t) = [p(t)]^{-1} \sum_{n=0}^{N} a_n h_n(t) \qquad (4.13)$$

Figure 8 shows a basic synthesis of the system. The approximate correlation function would be seen as a transient following the application of the impulse.

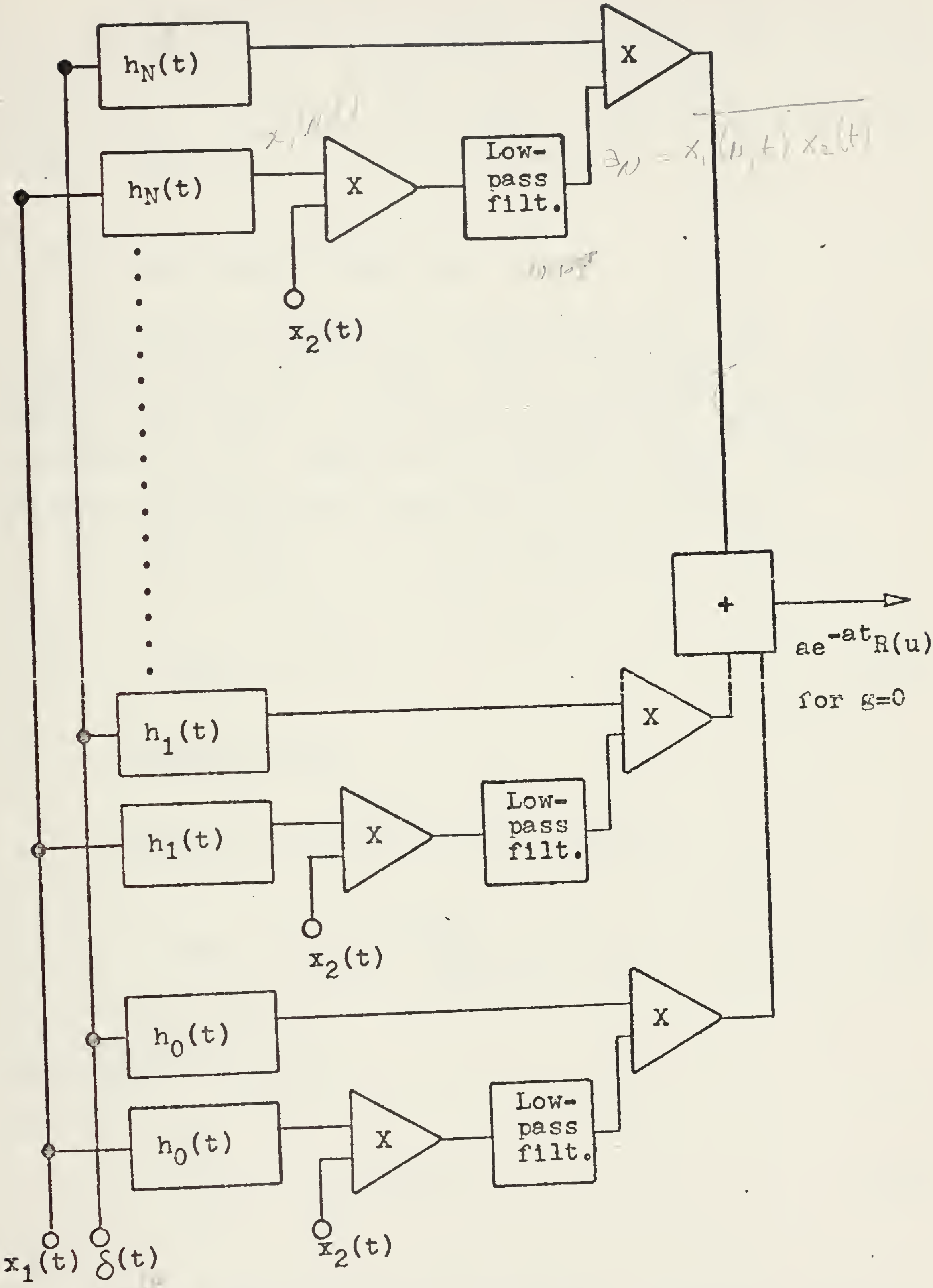


Figure 8: Correlator System without Time Delay

A convenient system results if in fact the pulsing can be carried out repetitively at intervals that are long compared with the filter time constants, and yet sufficiently frequently to give a steady trace on a CRT screen. [Ref. 1]

B. TWO REALIZABLE SYSTEMS. [Ref. 1,2]

By properly choosing the weight function p(t) and the parameter "g" two systems can be realized so that the impulse responses of the filters satisfy the definition of orthogonality as expressed by equation (4.08). This is so because $h_n(t)$ and $\theta_n(t)$ depend only on p(t) and "g".

For both systems, the weight function was chosen as:

$$p(t) = ae^{-at} \tag{4.14}$$

1. System I: g=0.

When g=0, equation (4.10) becomes:

$$R(t) = [p(t)]^{-1} \sum_{n=0}^{\infty} a_n h_n(t) \tag{4.15}$$

where $h_n(t)$ reduces to:

$$h_n(t) = \theta_n(t)[p(t)] \tag{4.16}$$

The Laplace transfer function of an RC low-pass filter is:

$$H_L(s) = \frac{a}{s+a} = L[h(t)] \qquad (4.17)$$

where

$$a = 1/t_1 = 1/RC \qquad (4.18)$$

Now

$$L^{-1}[H_L(s)] = h_L(t) = ae^{-at} \qquad (4.19)$$

For a simple CR high-pass filter, the transfer function is:

$$H(s) = \frac{s}{s+\frac{1}{t_1}} = \frac{s}{s+a} \qquad (4.20)$$

Now if one low-pass cell and n high-pass cells are cascaded together but isolated by a buffer amplifier, the resultant circuit will have a transfer function in the s-domain expressed as:

$$H_N(s) = \frac{a}{s+a} \cdot \frac{s^n}{(s+a)^n} = \frac{as^n}{(s+a)^{n+1}} \qquad (4.21)$$

Figure 9 shows such a circuit. The inverse Laplace transform of equation (4.21) is:

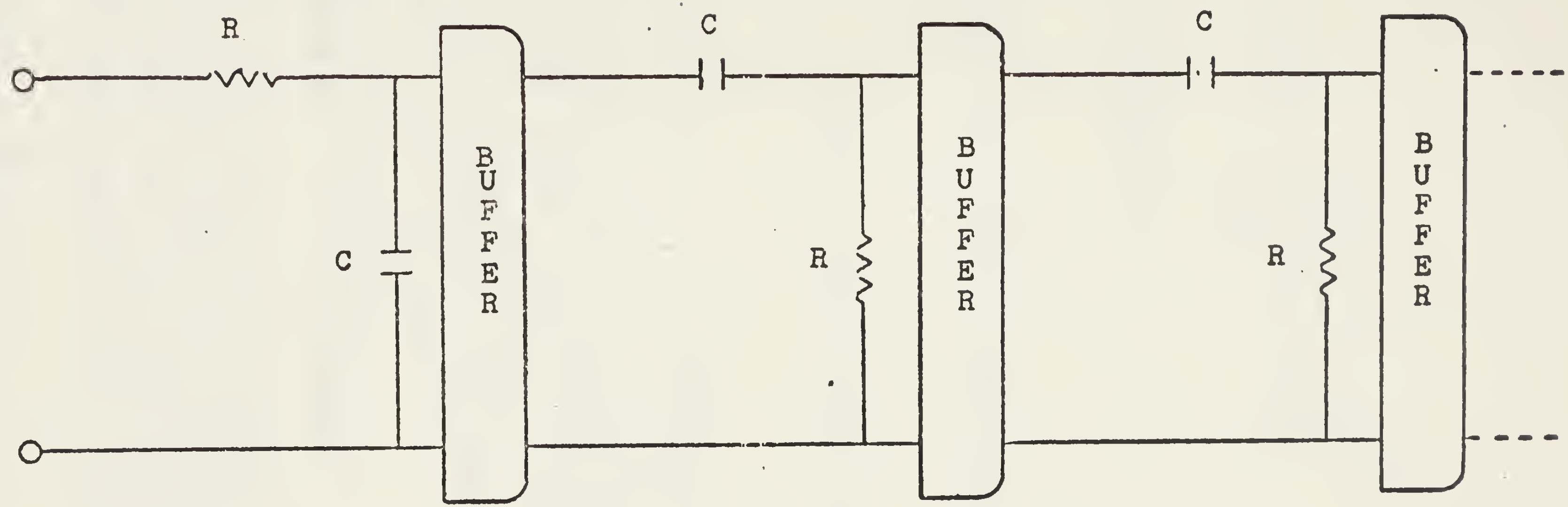


Figure 9: Orthogonal Filter for g=0.

$$h_n(t) = a\frac{d^n}{dt^n} \cdot \frac{1}{n} t^n e^{-at} \tag{4.22}$$

which can be written as

$$h_n(t) = ae^{-at} \sum_{k=0}^{n} [n!/(n-k)!][(-at)^k/k!\,k!] \tag{4.23}$$

$$= ae^{-at} L_n(at) = p(t) L_n(at) \tag{4.24}$$

where the product is a set of Laguerre functions and $L_n(at)$ is defined as a set of Laguerre polynomials.

It can be seen that equation (4.24) is of the same form of equation (4.08) where $L_n = \theta_n$ and so the circuit of figure 9 is by definition an orthogonal filter and can be used to realize the correlator.

2. <u>System II: g=1/2. (symmetry).</u>

The realization of this filter is presented briefly here for completness since the experimental work was carried out with filters of the first kind (g=0).

For g=1/2 equation (4.10) reduces to:

$$R(t) = \sum_{n=0}^{\infty} a_n h_n(t) \tag{4.25}$$

and $h_n(t)$ reduces to:

$$h_n(t) = \theta_n [p(t)]^{1/2} \tag{4.26}$$

Figure 10 shows the circuit diagram of a filter that can also satisfy equation (4.08). In this case:

$$H_n(s) = \frac{\frac{a}{2}}{s+\frac{a}{2}} \cdot \left[\frac{s-\frac{a}{2}}{s+\frac{a}{2}}\right]^n \tag{4.27}$$

where

$$a/2 = 1/RC \tag{4.28}$$

and its inverse Laplace transform is:

$$h(t) = \frac{ae^{-a/2t}}{2} L_n(at) \qquad 4.29)$$

which is identical to equation (4.24) except for a constant factor.

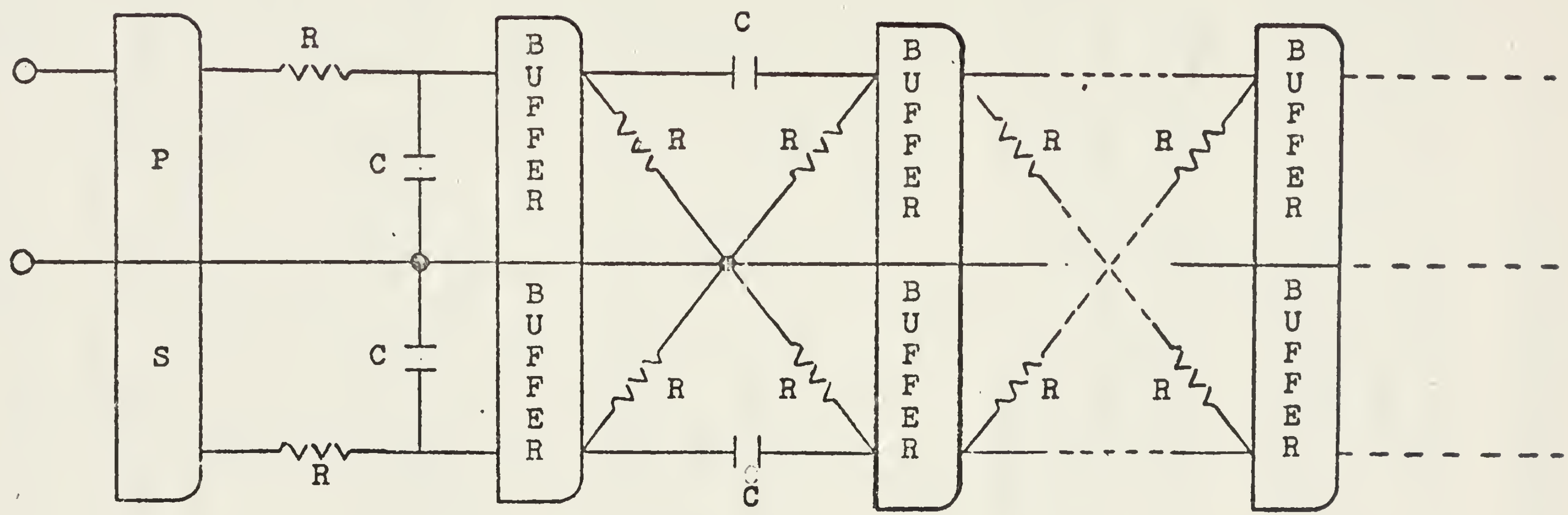


Figure 10: Orthogonal Filter for g=1/2.

V. EXPERIMENTAL PROCEDURE.

The practical work was carried out in two parts. First, the correlator system shown on figure 11 was simulated in the computer using the DSL program, and second, experiments were conducted on an actual correlator built in the laboratory.

In both cases, the results of the autocorrelation of a sine wave of 155 hz were compared. The practical results obtained were very much as predicted by the simulation.

A detailed explanation of both the simulation and the experiment will be shown in the following pages.

A. NATURE OF THE PROBLEM.

Figure 11 represents the block diagram of the proposed correlator. In this case, economy, availability of parts, the accuracy of the results and the area in which the correlator could be used as a practical device, were the guidelines for the design and construction of the correlator shown.

Minimum cost was one of the main objectives of the design work. For this reason, the orthogonal filters were designed without the buffer amplifiers but this requirement limited the range of values of the resistors and capacitors necessary to realize the linear filters. Also, the values of the resistances and capacitances used were fixed which presented the problem of restricting the correlator to operate in a very narrow band of frequencies. Fortunately, since the filters are linear, the possibility exists of increasing the frequency of operation by varying the time constant $t = RC$.

The low-pass filters, shown after the multipliers, were designed as simple RC circuits.

All of the above considerations mean that several

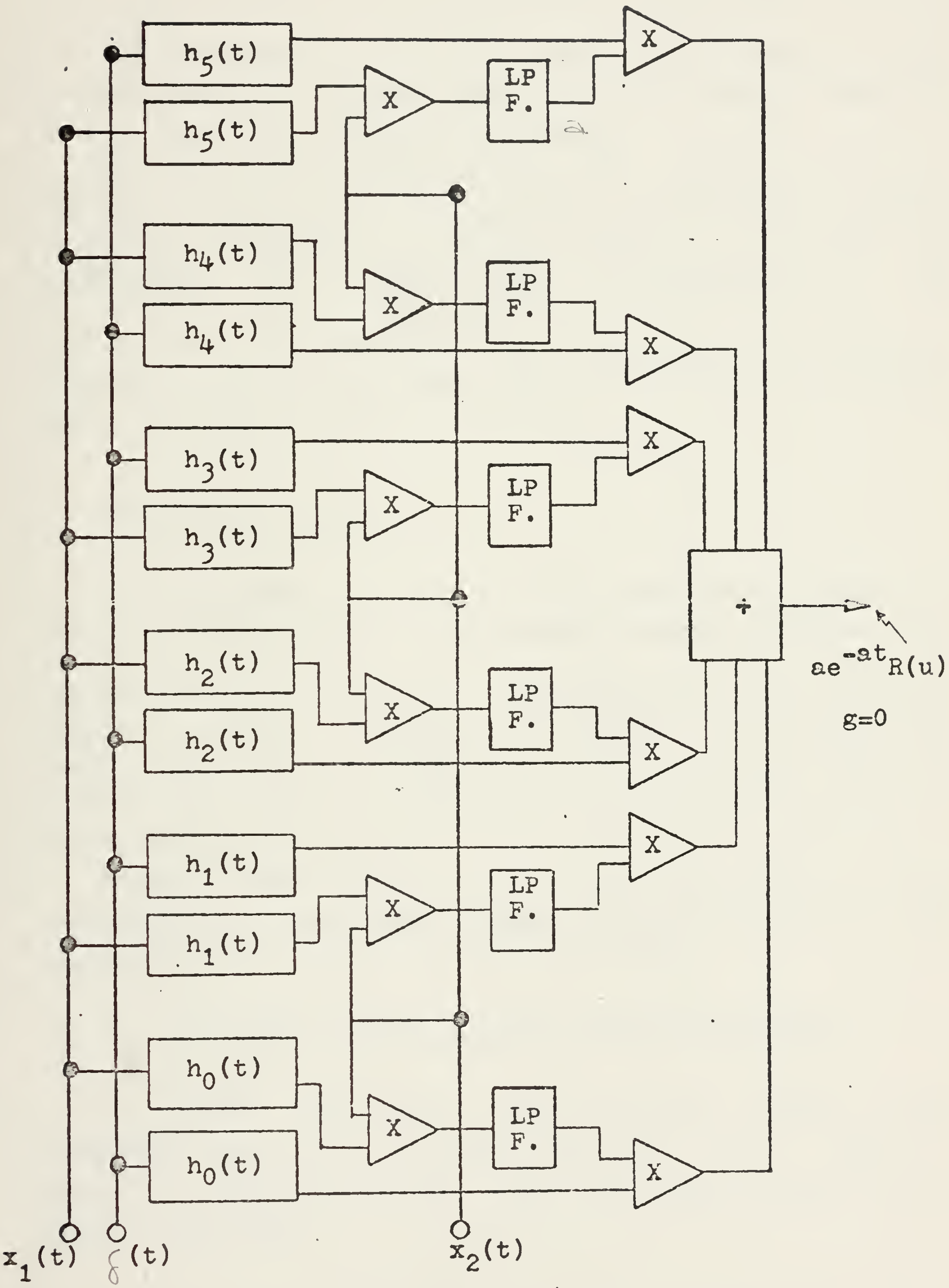


Figure 11: Block Diagram of practical Correlator.

non-linearities were introduced in the system. Nevertheless the final results were consistently accurate for practical purposes.

The correlator was also restricted to operate at low frequencies which together with its narrow-band operation made it useful in underwater detection.

The next problem that had to be worked out was to determine the frequency of operation of the train of pulses necessary to produce the transient needed to generate the coefficients a_n. After some considerations on the limitations of the correlator it was decided to use the same frequency of the sine wave. The pulses then were in phase with the positive peaks of the wave.

B. LINEAR-FILTER DESIGN.

It can be seen in figure 11 that the total number of filters used to generate the coefficients a_n was equal to 6 for the orders n = 0,1,...5. Mathematically, an infinite number of linear filters are required to generate the coefficients, but the results obtained by using only six filters were very much within the theoretical considerations and computer predictions.

Higher order filters were not used because of the availability of components and the characteristics of the available pulse generator and oscilloscope.

1. Orthogonal Filters without Buffer Amplifiers.

Buffer amplifiers have ideally infinite input impedance, zero output impedance and unity gain. Figure 9 shows an orthogonal filter which uses buffer amplifiers to isolate the cells.

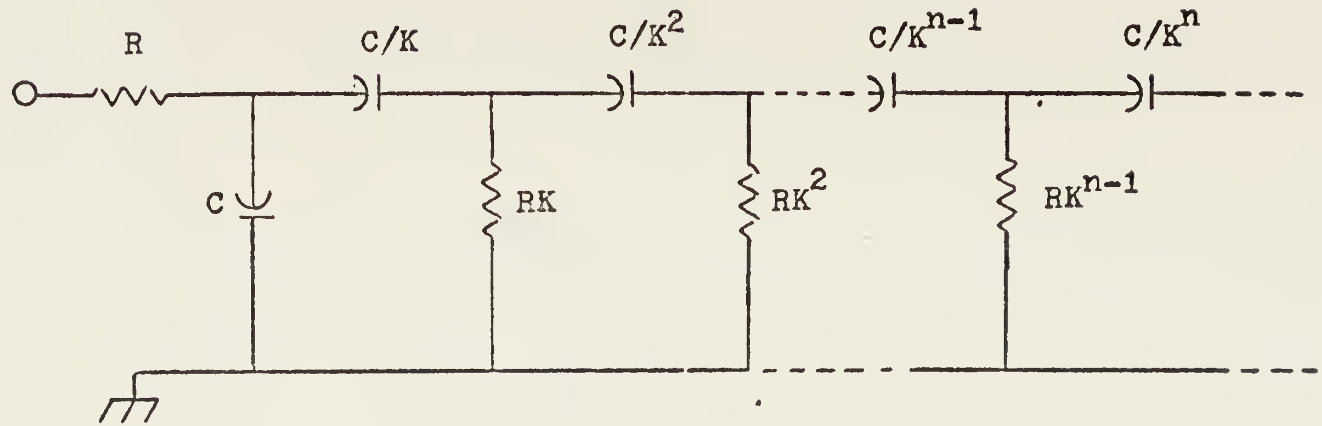


Figure 12: Orthogonal Filter without Buffer Amplifier.

Figure 12 shows a method which can also effectively isolate the cells and at the same time eliminates the buffer amplifiers. The method consists in introducing a factor of isolation k which multiplies the values of the resistors and capacitors in such a way as to maintain the time constant the same throughout the cells. [Ref. 1]

The maximum value of k that can be used depends upon the number of cells and the available values of resistors and capacitors. It was found that for this waork, a factor of isolation of over 10 would require values much too big for the resistors and too small for the capacitors. For k less than 5, the approximation of the ideal impulse response was not accurate enough. A value of 5 was finally used and the results are presented in figures 19,20, and 21.

The number of cells of the filters depends upon the degree of the Laguerre polynomial. The low-pass RC cell represents the Laguerre Polynomial of degree 0 (n=0). The degree is increased by cascading high-pass RC cells. The maximum number of cells that could be used without introducing appreciable distortion was found to be 6 (n=5): one low-pass cell and five high-pass cells.

2. Details of circuitry.

A simplified schematic diagram of the orthogonal filter for n=5 is shown in Figure 13. Theoretically:

$$R_n = k^n R_0 \qquad C_n = C_0/k^n$$

and the following values were chosen:

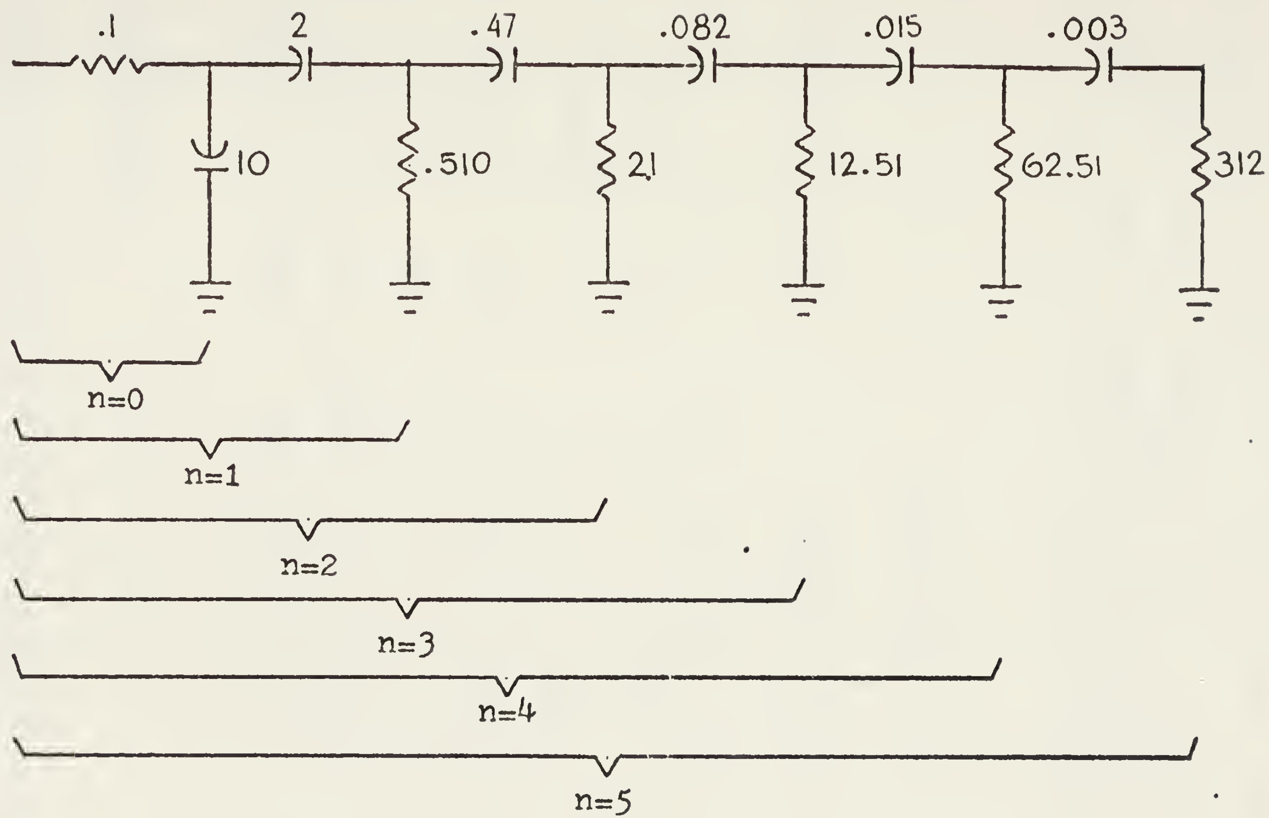


Figure 13: Schematic Diagram of Orthogonal Filter (n=5).

$R_0 = 100ohm \quad C_0 = 10\ MF$

$a = 1/RC = 1000 \quad k = 5$

Table I shows the actual values used together with the theoretical values.

The filters required for the lower orders can be obtained by simply eliminating one high-pass cell for each order. Figures 19, 20 and 21 shows the actual results.

TABLE I.

Theoretical Values				Actual Values.				
	R		C	R	K_R	C	K_C	a
R_0	.1K	C_0	10MF	.1K		10MF		1000.00
R_1	.5K	C_1	2MF	.51K	5.1	2MF	5.0	980.39
R_2	2.5K	C_2	.40MF	2.1K	4.6	.47MF	4.6	1013.17
R_3	12.5K	C_3	.08MF	12.51K	5.0	.082MF	4.9	974.83
R_4	62.5K	C_4	.016MF	62.51K	5.0	.02MF	4.7	953.65
R_5	312.5K	C_5	.0032MF	312K	4.8	.0039MF	5.1	931.46

C. MULTIPLIERS.

Integrated circuit multipliers were used to realize the correlator. In this case, the Intersil 8013A was used which is a four quadrant analog multiplier and whose output is proportional to the algebraic product of two input signals.

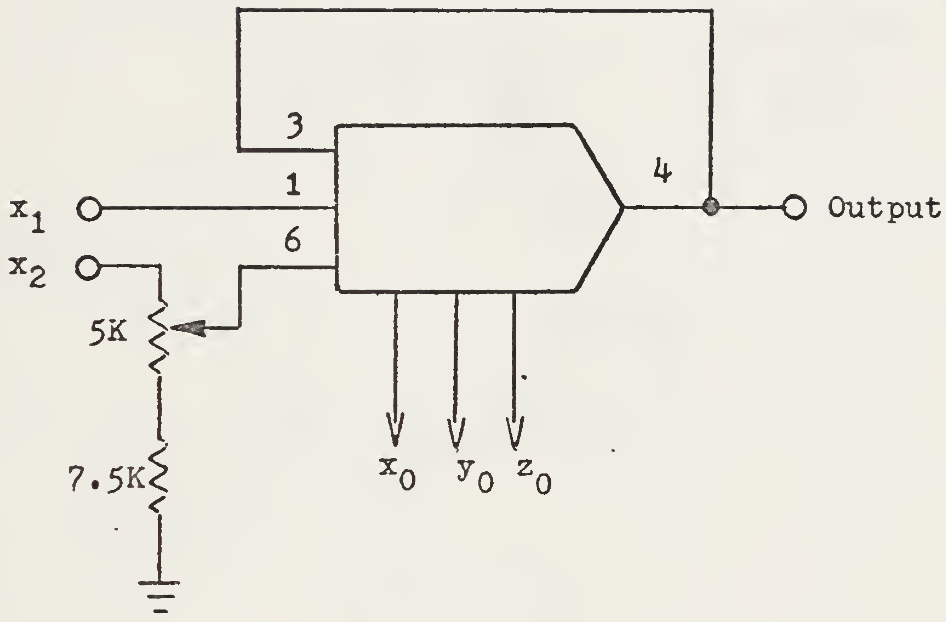


Figure 14: Block Diagram of Multiplier.

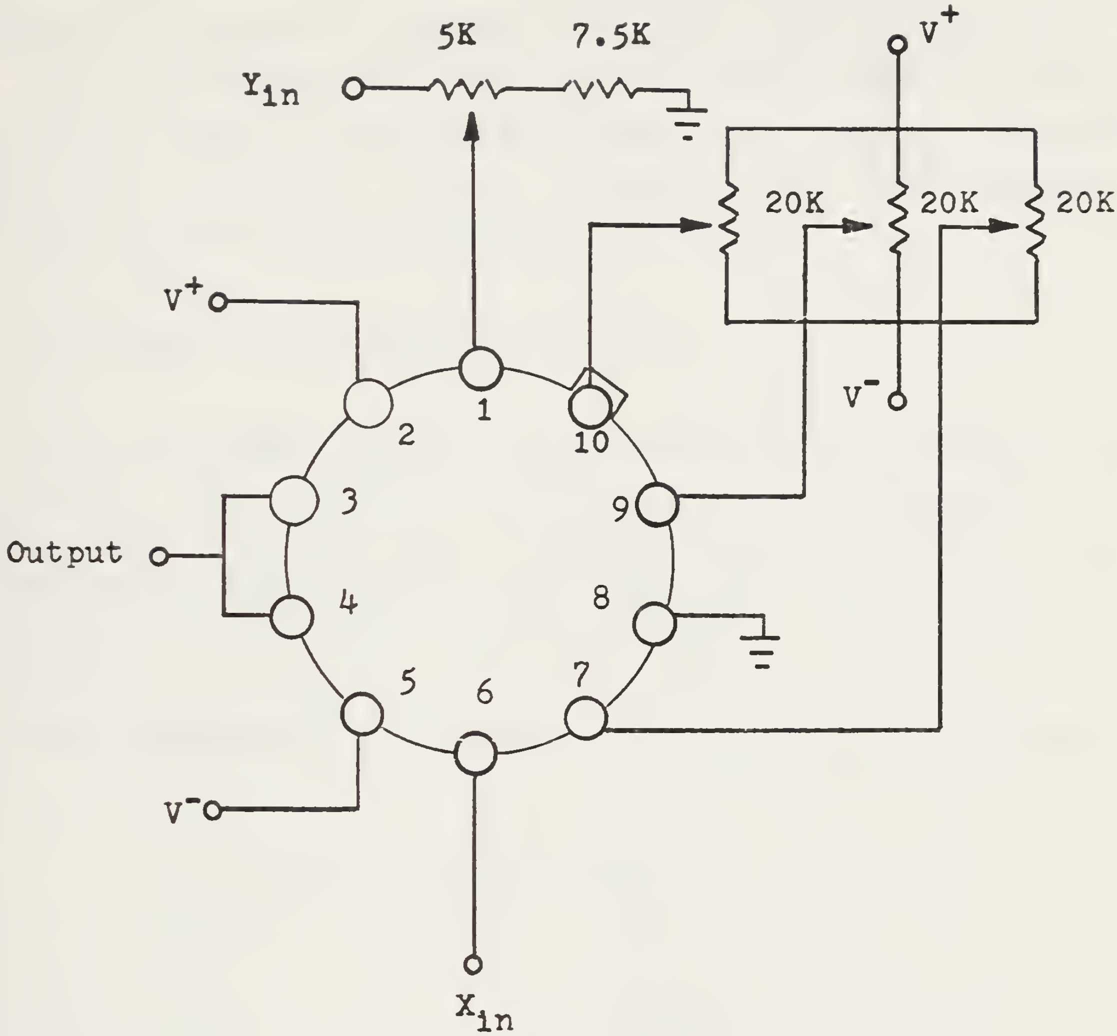


Figure 15: Connection Diagram of Multiplier.

The multipliers can operate with standard ± 15 volts supplies, and have a maximum dissipation power of 500 mW.

The block diagram of the multipliers is shown in figure 14 while figure 15 shows the connection diagram together with the trimming circuit necessary for gain accuracy, offset voltage and feedthrough performance.

D• LOW-PASS FILTER DESIGN• [Ref. 4]

The low-pass filters were designed as simple R-C low-pass cells with a cut-off frequency of about 600 Hz. More accurate filters can be designed, such as second order Butterworth filters, but in this case, the simple R-C low-pass filters were accurate enough for practical purposes.

The designed of the filters was carried out as follows:

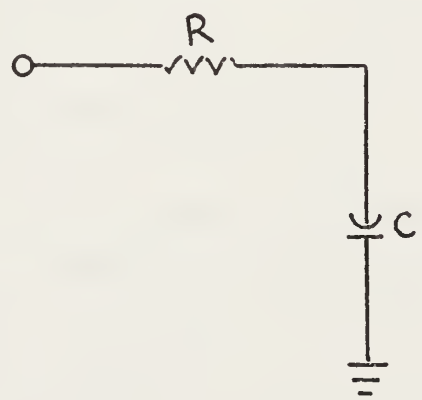


Figure 16: Low-pass Filter Design.

$$f_H = \frac{1}{2\pi RC} \qquad (5.01)$$

If R is chosen to be 510 Ohms and C equal to 0.47 MF,

then the cut-off frequency would be:

$$f_H = \frac{1}{2\ (510)(0.47\times10^{-6})} = 663.97\ \text{Hz} = 664\ \text{Hz} \tag{5.02}$$

For the computer simulation, a real-pole transfer function is required. This transfer function is given by:

$$H_{LP}(s) = \frac{1}{Ps+1} = \frac{1}{RCs+1} \tag{5.03}$$

where $P = RC = 510(0.47\times10^{-6}) = 2.397\times10^{-4} = 2.4\times10^{-4}$

and P is a parameter required for the real-pole transfer function computer block.

E. THE ADDER AMPLIFIER. [Ref. 4]

The output of the adder or summing amplifier of figure 17 is a linear combination of the input signals. This arrangememt was used to construct the adder required in the last stage of the correlator.

As will be explained in the next section, the output of the amplifier is not the true correlation function but the product of the correlation with the weight function used to realize the linear filters. Nevertheless, the shape of the output can very well be used for signal identification since it is a characteristic of the signal being correlated.

Figure 18 is the circuit diagram of the correlator.

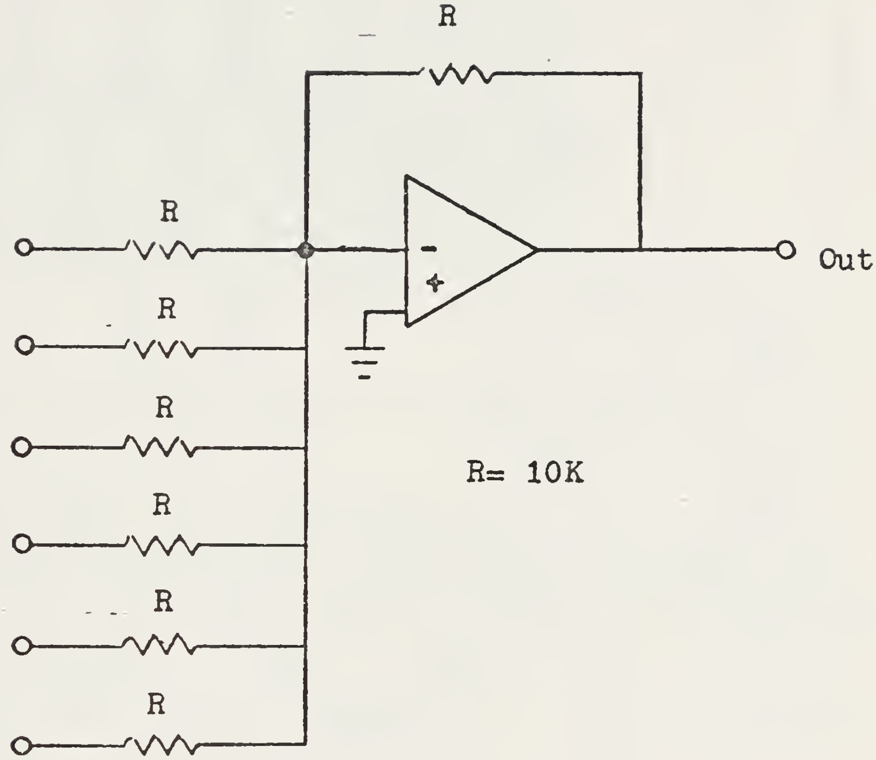


Figure 17: Diagram of Adder Amplifier.

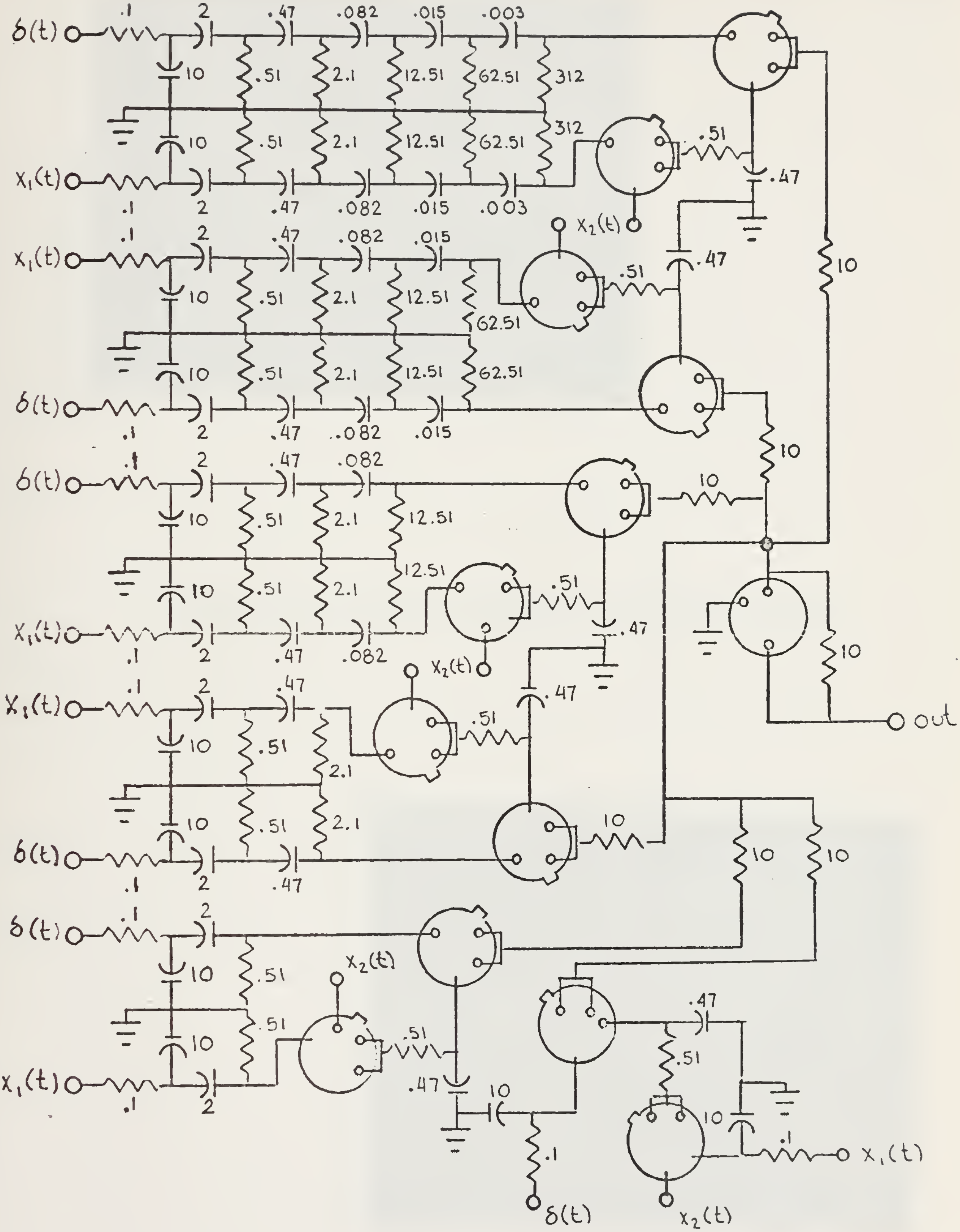
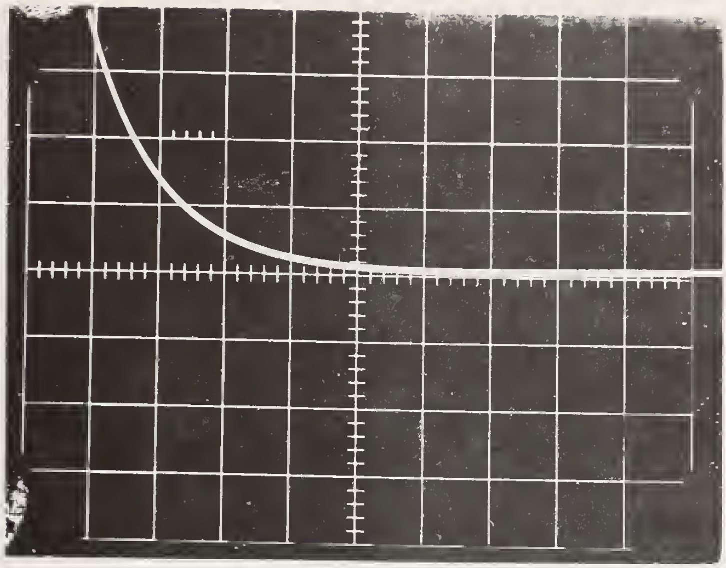


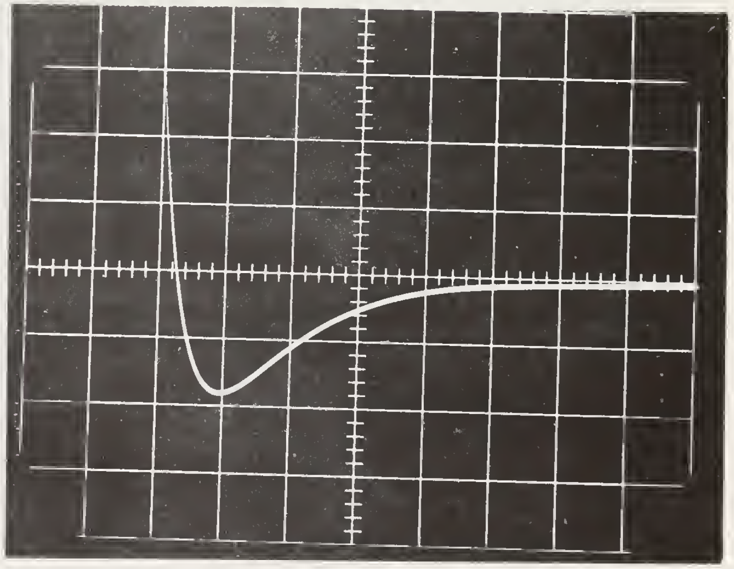
Figure 18: Correlator Circuit Diagram.

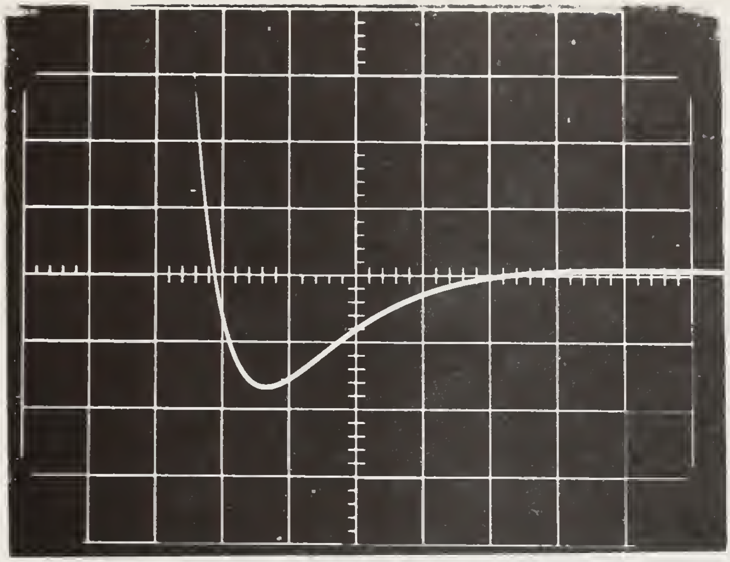


n=0

Figure 19: Impulse Response of Linear Filters.

n=1

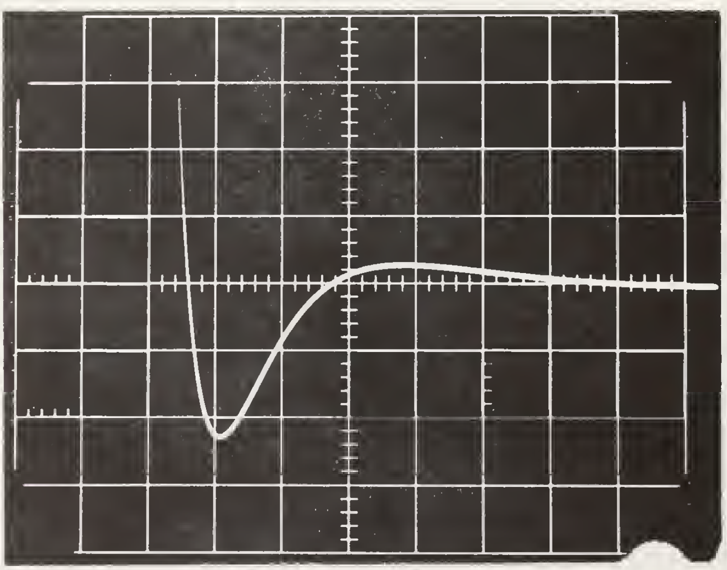


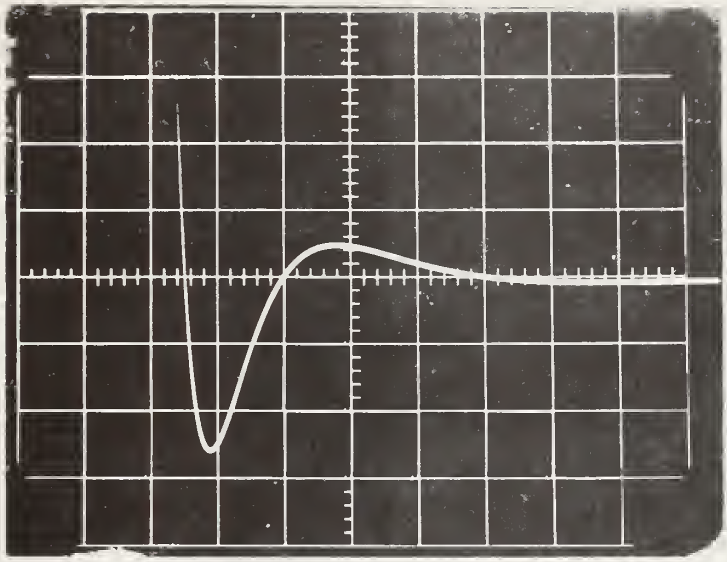


n=2

Figure 20: Impulse Response of Linear Filters.

n=3

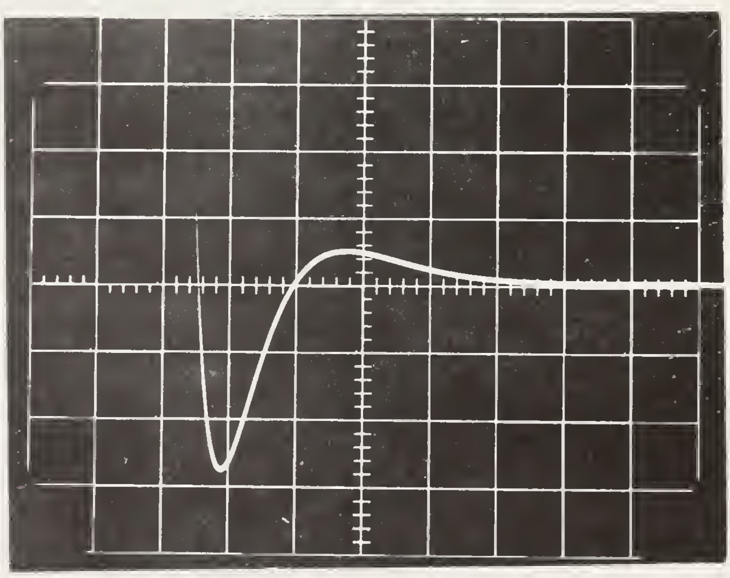




n=4

Figure 21: Impulse Response of Linear Filters.

n=5



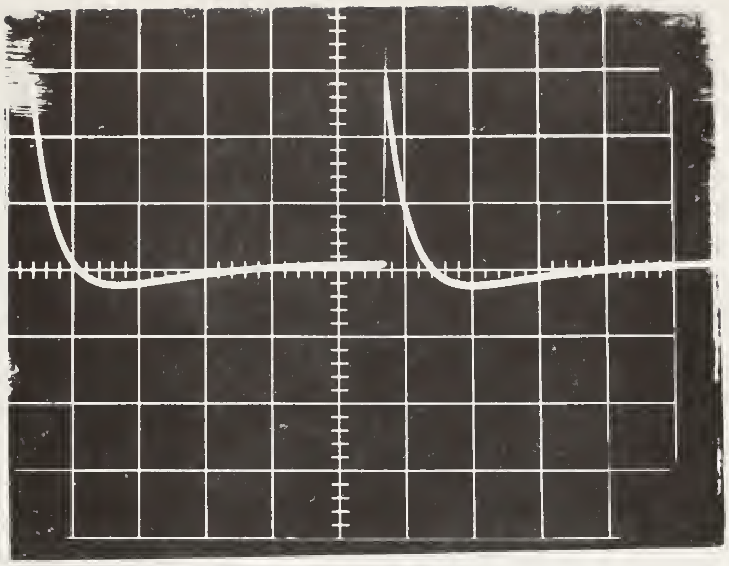


Figure 22: Practical Correlator Output.

VI. COMPUTER SIMULATION.

Figure 22 shows the diagram used to simulate the correlator on DSL (Digital Simulation Language) together with the name of the variables used for the program which appears in appendix A.

Since the linear filters used to build the correlator were those for which g=0, the actual output of the correlator is not the true correlation function but the correlation function multiplied by the inverse of the weight function p(t). In the case of the autocorrelation of the sine wave, the output of the correlator which is shown in Figure 24, is:

$$R'_{xx}(u) = ae^{-at} R_{xx}(u) \qquad (6.01)$$

In order to get the true autocorrelation function of the sine wave shown in Figure 23 $R'_{xx}(u)$ has to be multiplied by the function e^{at}/a shown on figure 25 This function is an exponential which increases very rapidly and overflows the computer results for times beyond those shown on the figure. Scaling can be used to compensate the overflow and increase the time of observation of the autocorrelation function. Thus:

$$R_{xx}(u) = \frac{e^{at}}{a} R'_{xx}(u) \qquad (6.02)$$

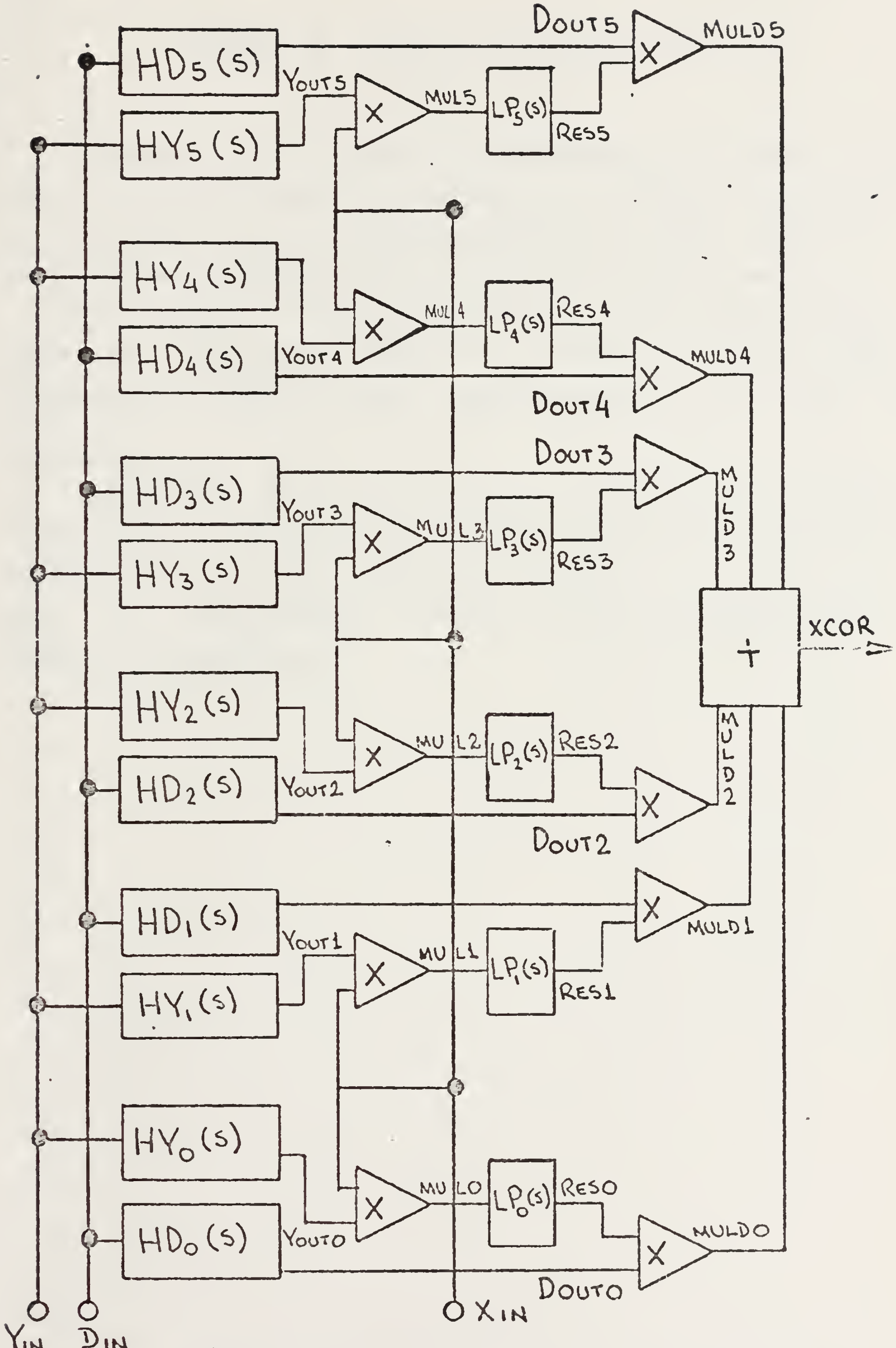


Figure 23: Simulation of Correlator.

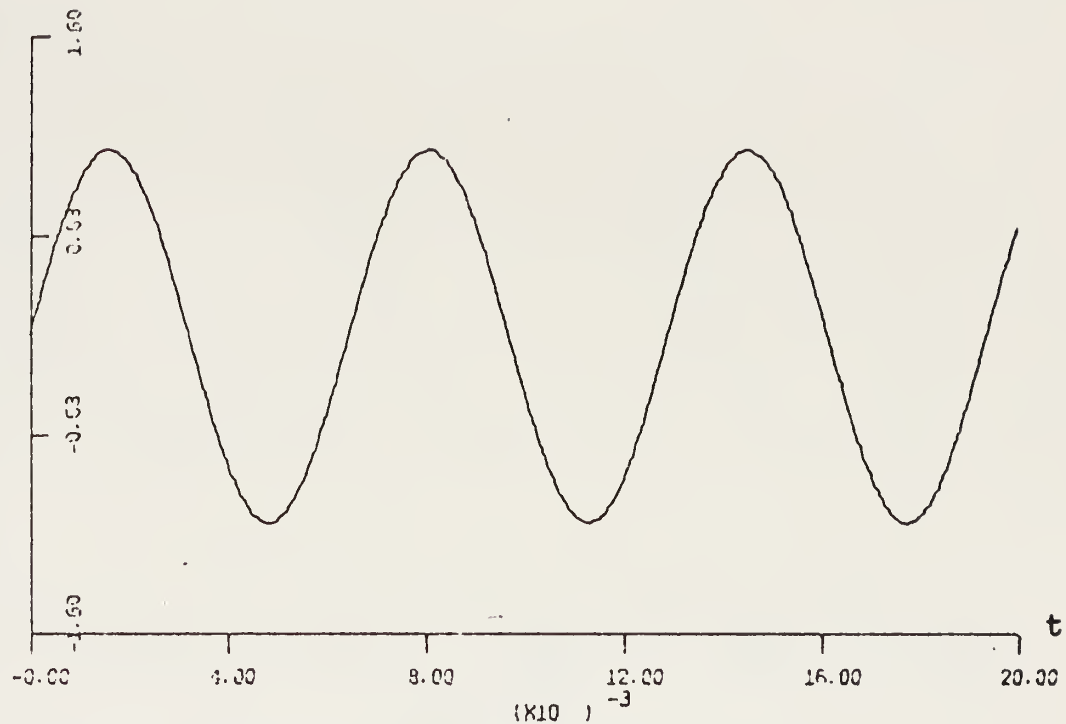
or

$$\text{Figure 26} = \text{Figure 24} \times \text{Figure 25}$$

Figures 28 to 33 show the response of the linear filters for a train of impulses. They can be compared to Figures 19 to 21 which show the impulse response of the filters. The pictures were obtained from the experimental work.

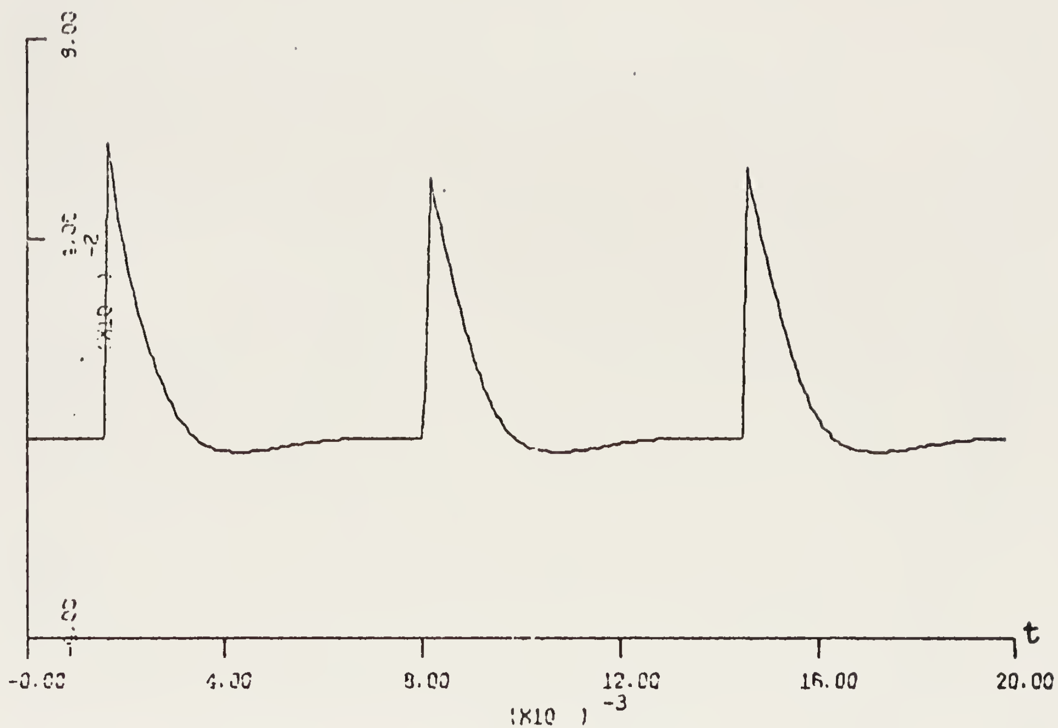
The response of the filters are of the same form as the Laguerre Functions shown on Figure 27 and which are necessary to generate the coefficients a_n of the series expansion.

Figure 22 shows the output of the experimental correlator. This figure can be compared to Figure 25 which represents the output of the simulation. It can be seen that the experimental results were in agreement with the computer predictions.



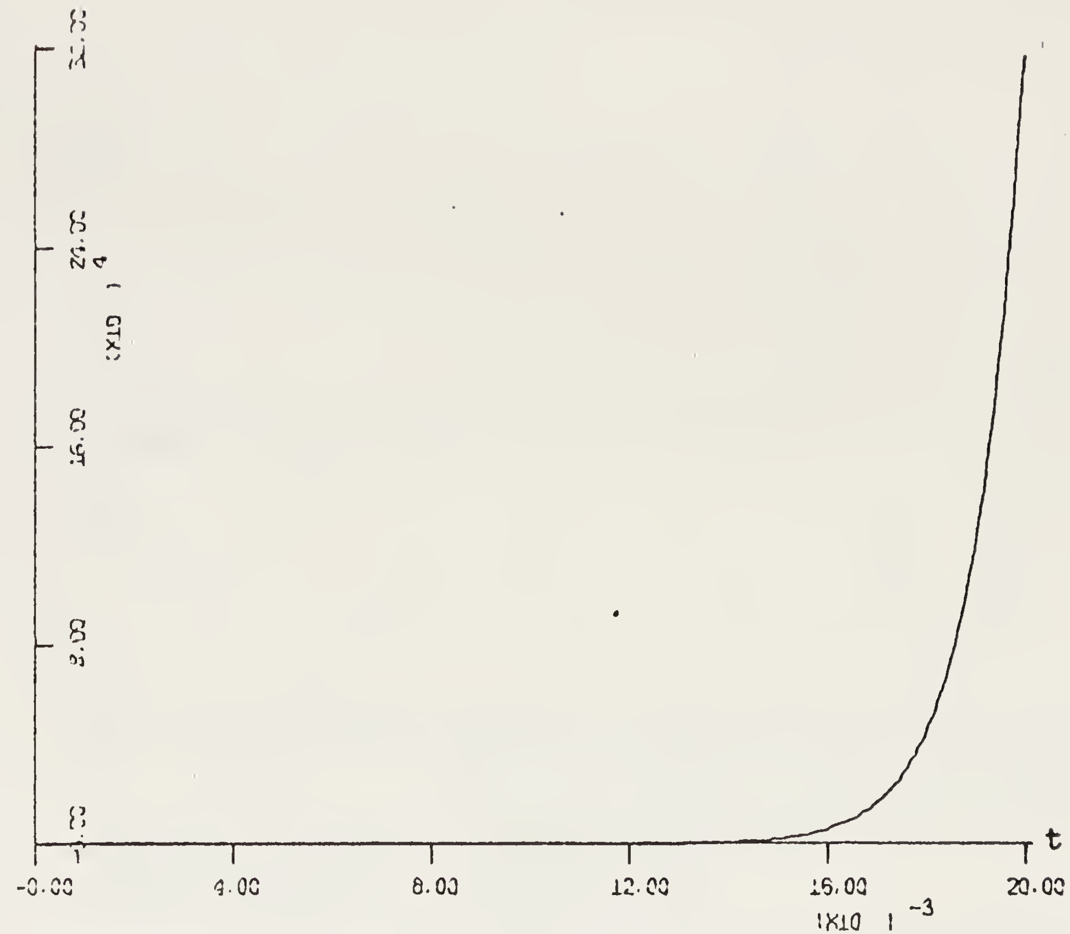
XSCALE= 4.00E-3 UNITS/INCH
YSCALE=1.07 UNITS/INCH

Figure 24: Correlator input: Sine Wave (155 Hz).



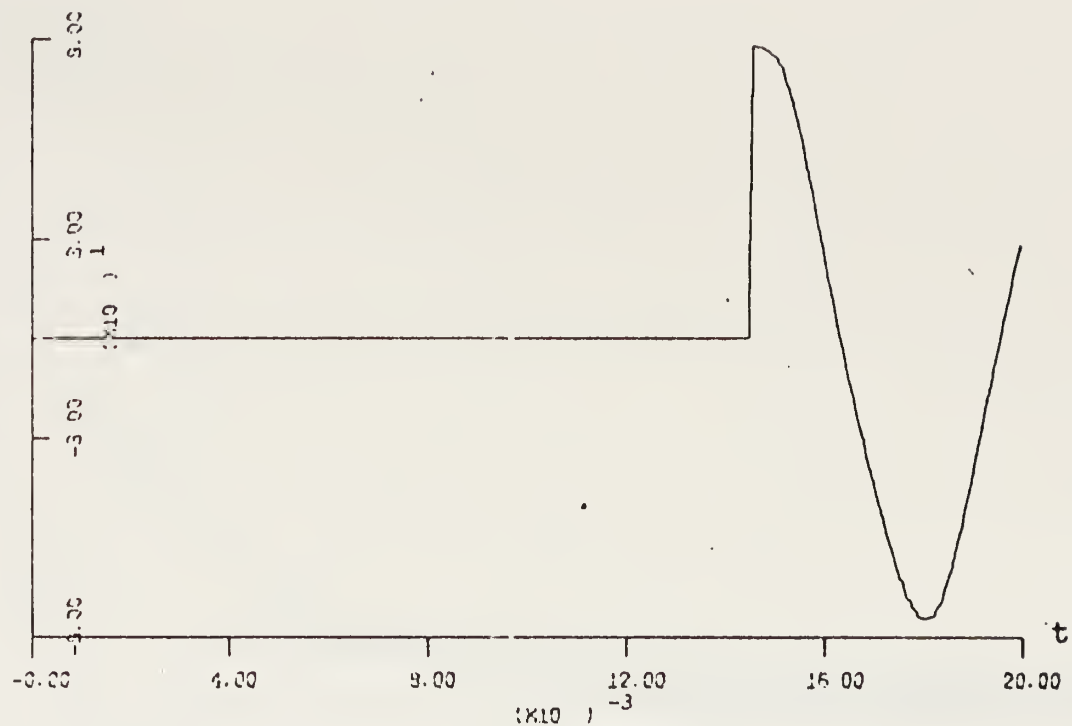
XSCALE= 4.00E-3 UNITS/INCH
YSCALE=0.04 UNITS/INCH

Figure 25: Correlator Output: $R'_{xx}(u)$.



XSCALE= 4.00E-3 UNITS/INCH
YSCALE=80000.00 UNITS/INCH

Figure 26: Exponential Function: e^{at}/a.



XSCALE= 4.00E-3 UNITS/INCH
YSCALE=60.00 UNITS/INCH

Figure 27: Autocorrelation Function of a Sine Wave.

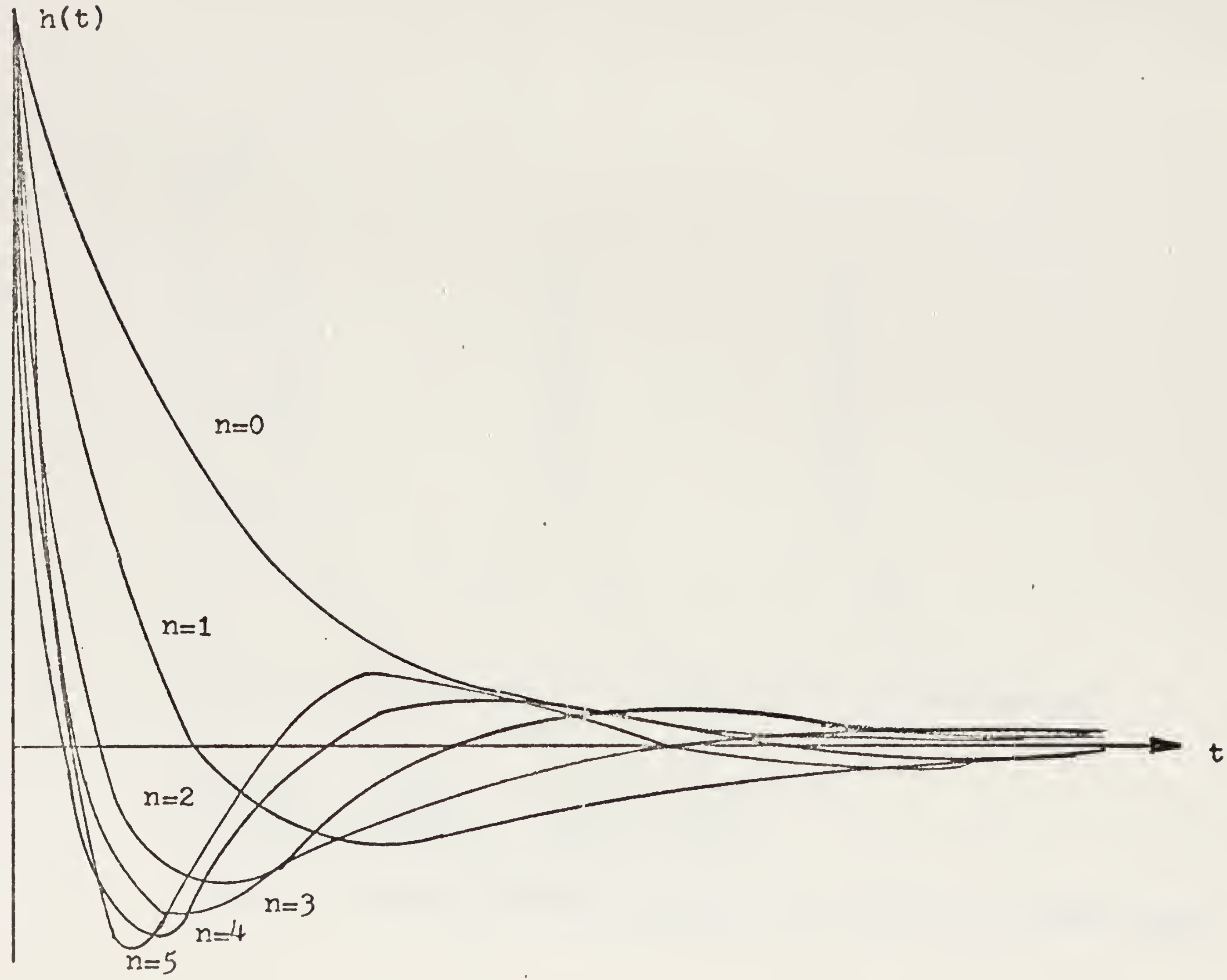


Figure 28: Laguerre Functions.

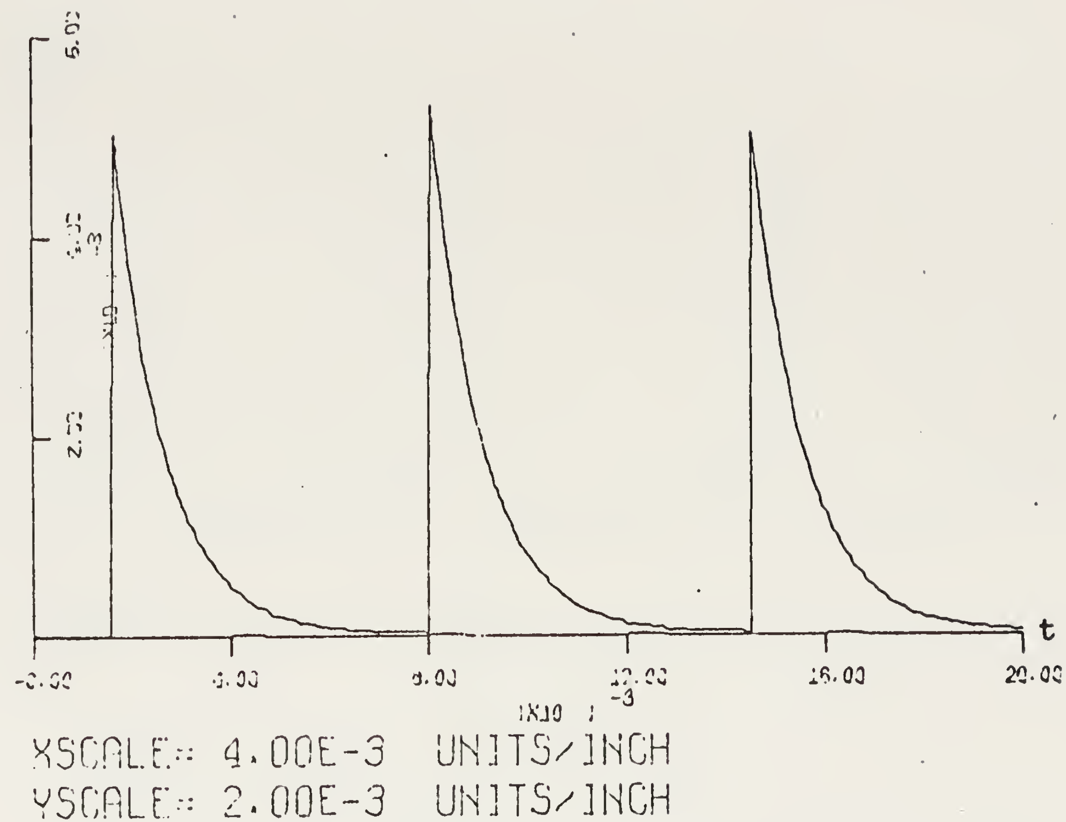


Figure 29: Linear Filter Response to a Train of Pulses (n=0)

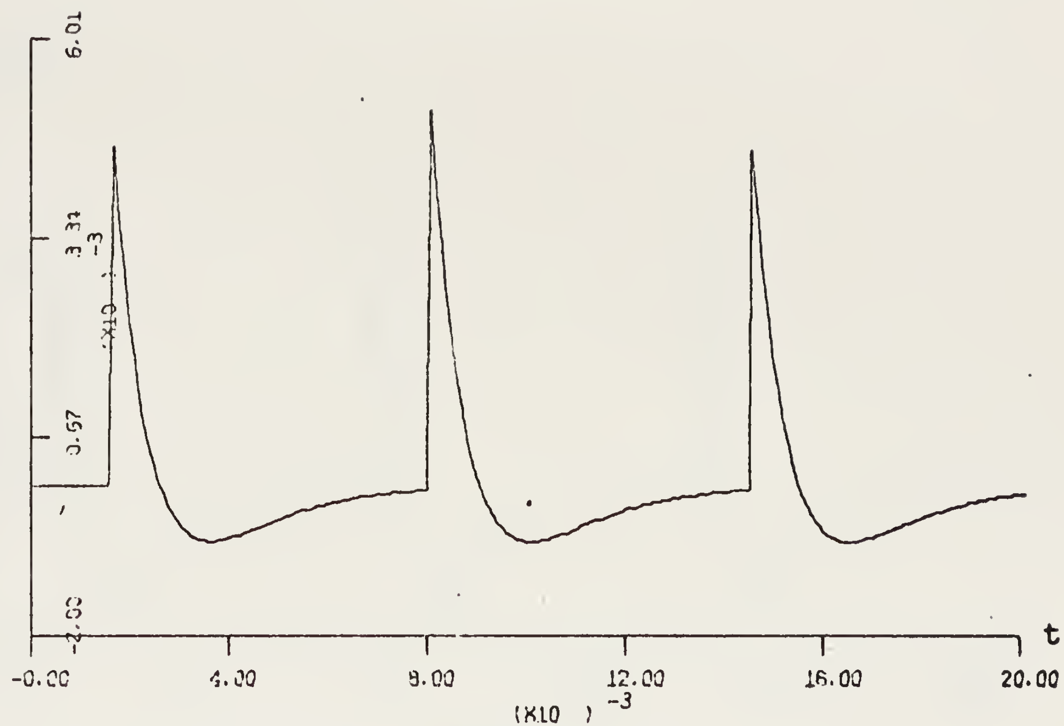
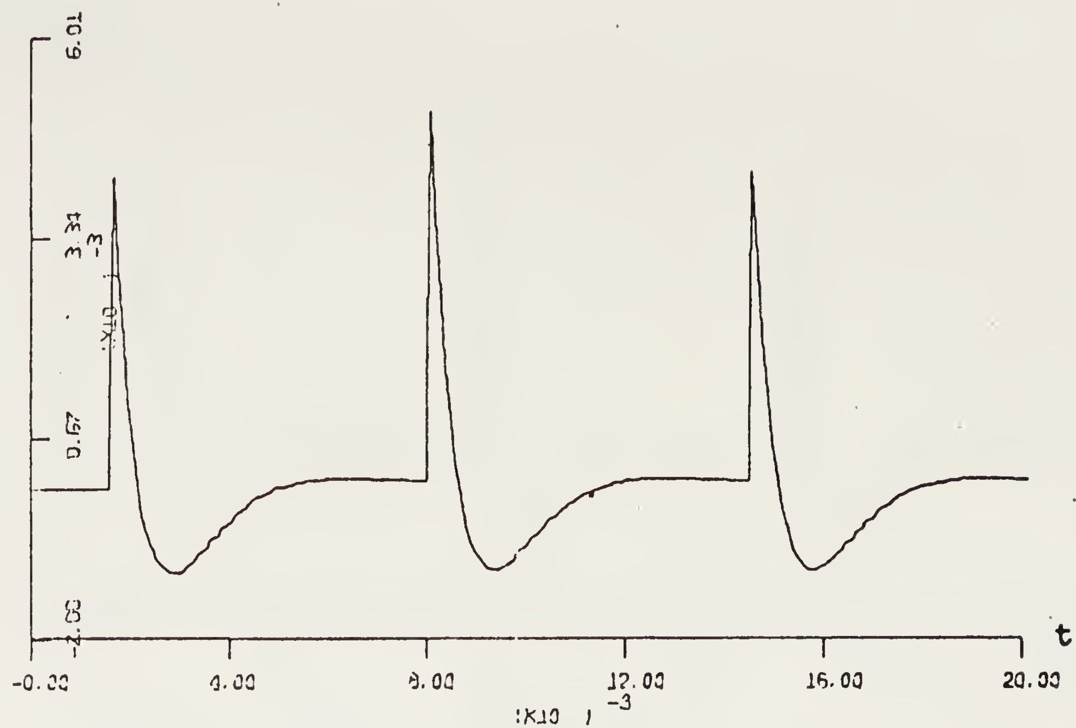


Figure 30: Linear Filter Response to a Train of Pulses (n=1)



XSCALE= 4.00E-3 UNITS/INCH
YSCALE= 2.67E-3 UNITS/INCH

Figure 31: Linear Filter Response to a Train of Pulses (n=2)

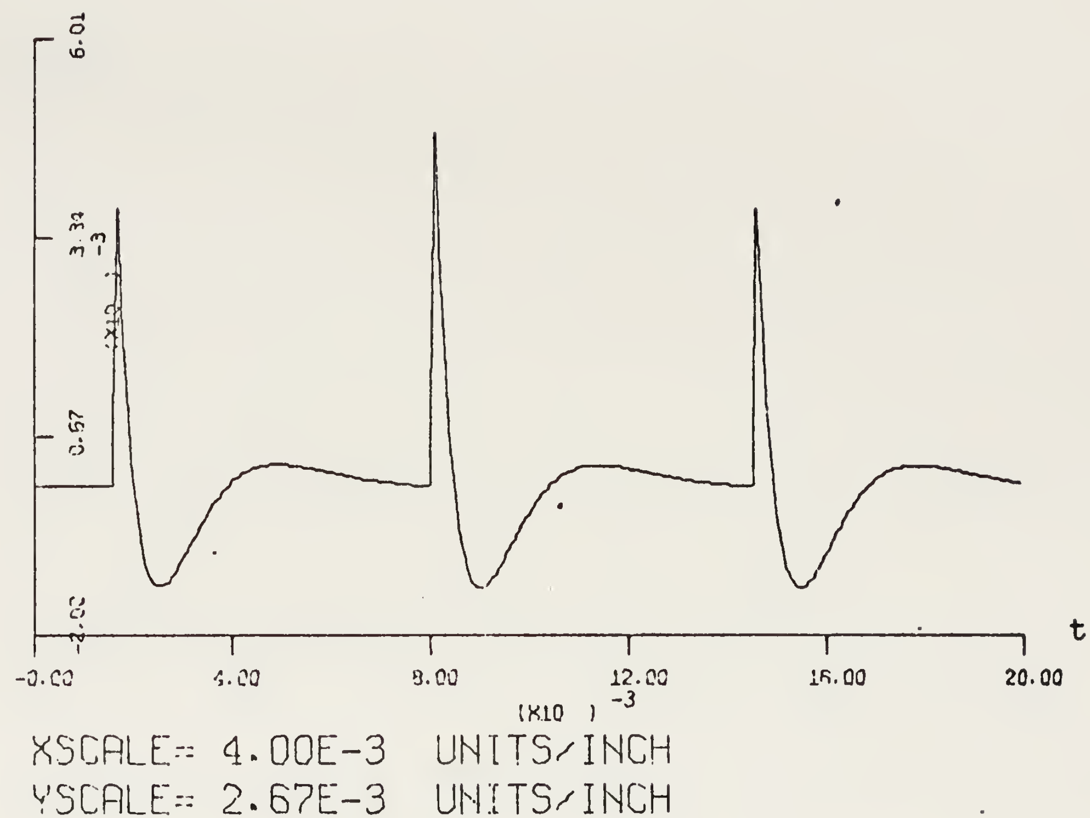


Figure 32: Linear Filter Response to a Train of Pulses (n=3)

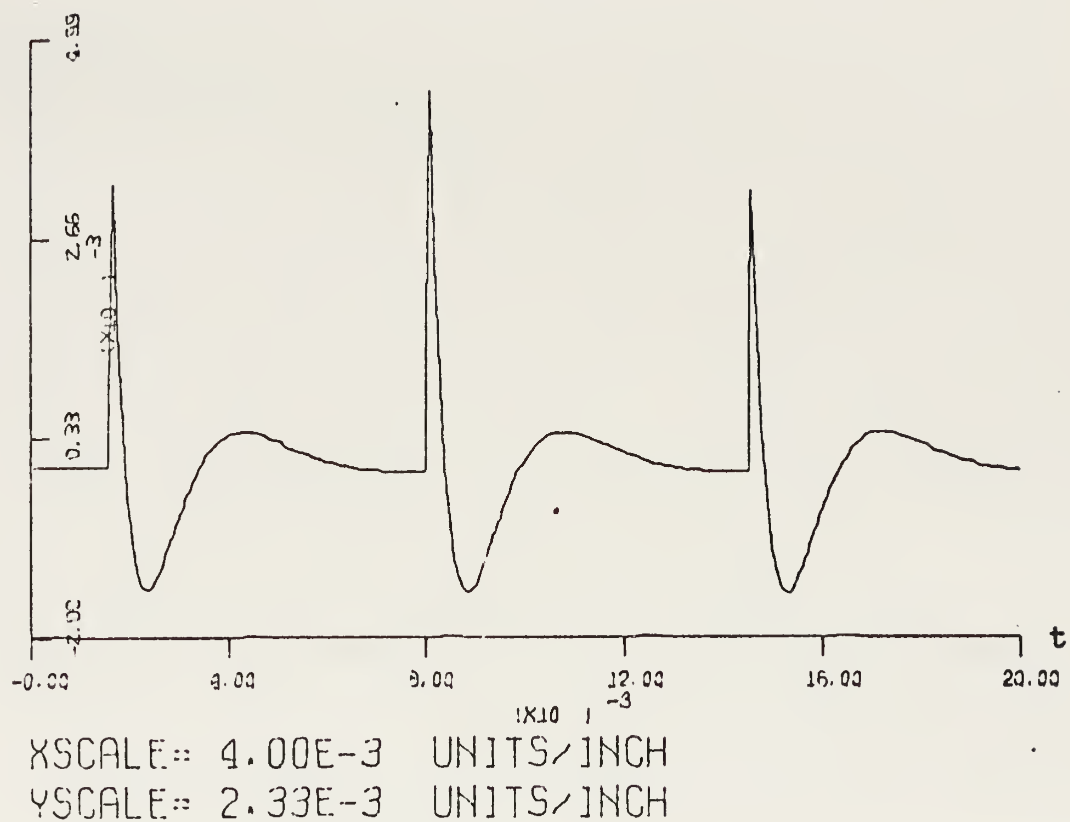


Figure 33: Linear Filter Response to a Train of Pulses (n=4)

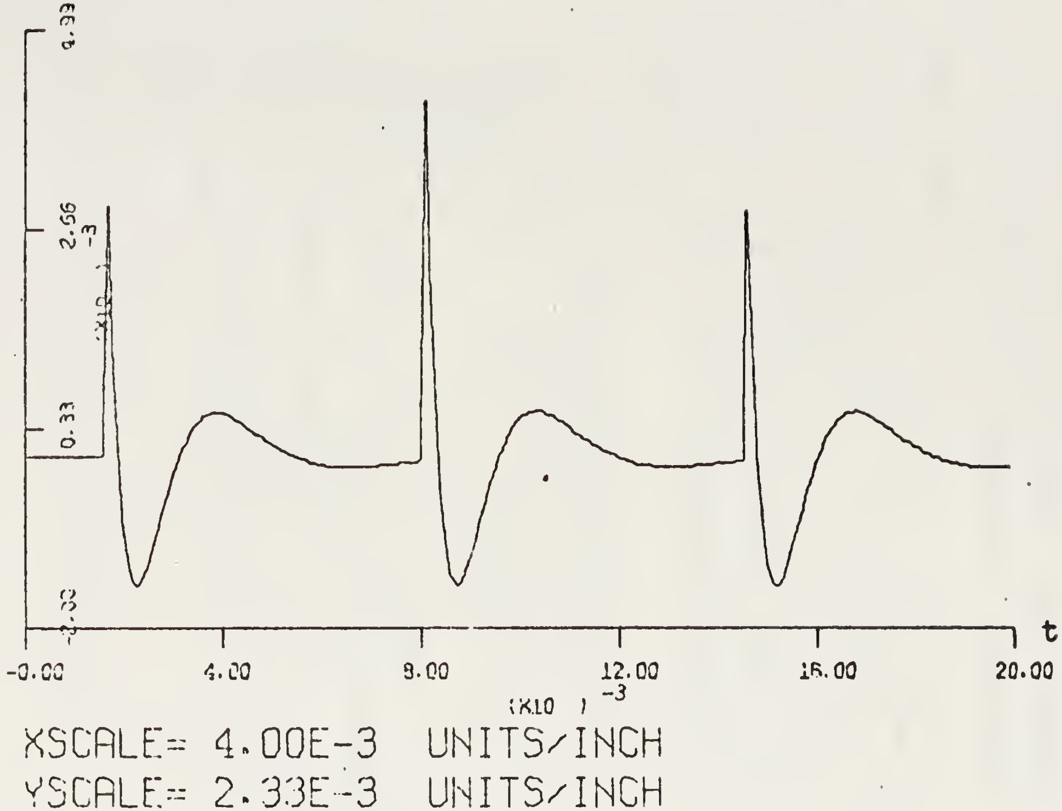


Figure 34: Linear Filter Response to a train of pulses (n=5)

VII. CONCLUSIONS.

The orthogonal filters were designed in such a way as to eliminate the buffer amplifiers which were required to isolate the cells. This requirement, together with the fact that the values of the resistances and capacitances used were fixed, restricted the correlator to operate in a narrow band of frequencies.

Some work can be done in this area in order to investigate the possibility of increasing the range of frequencies of operation of the correlator by means of variable resistances and capacitances which will vary the time constant t=RC. This is entirely possible because the filters are linear.

Also, further investigation may be required for the use of some other type of multipliers in such a way that they can be grouped as one unit instead of using 12 separated units as designed.

The actual output of the correlator is not the true correlation function but is an exponentially decayed correlation as shown in section VI. So, it would be worth investigating on the possibility of obtaining the true correlation function by the generation of an exponential of the type e^{+at}/a in order to compensate the exponentially decayed correlations obtained in this study.

SIMULATION OF CORRELATOR.

(DSL SUBROUTINE)

```
TITLE   CORRELATION ANALYSIS
*
*
INTEG   TRAPZ
INTGER  NPLOT
CONST   NPLOT=2
PARAM   PI=3.14159
PARAM   LPT=2.4E-04
PARAM   FREQ=50
PARAM   TETA=0.0
PARAM   ALFO=1000
PARAM   ALF1=990.2
PARAM   ALF2=997.85          Constant
PARAM   ALF3=992.1
PARAM   ALF4=953.7
PARAM   ALF5=931.46
*
*
INITIAL
*
*
        A1(1)=ALF1
        A2(1)=ALF2
        A3(1)=ALF3               Initial computations.
        A4(1)=ALF4
        A5(1)=ALF5
        B1(1)=1
        B1(2)=2*ALF1
        B1(3)=ALF1**2
        B2(1)=1
        B2(2)=3*ALF2
        B2(3)=3*ALF2**2
        B2(4)=ALF2**3
        B3(1)=1
        B3(2)=4*ALF3
        B3(3)=6*ALF3**2
        B3(4)=4*ALF3**3
        B3(5)=ALF3**4
        B4(1)=1
        B4(2)=5*ALF4
        B4(3)=10*ALF4**2
        B4(4)=10*ALF4**3
        B4(5)=5*ALF4**4
        B4(6)=ALF4**5
        B5(1)=1
        B5(2)=6*ALF5
        B5(3)=15*ALF5**2
        B5(4)=20*ALF5**3
        B5(5)=15*ALF5**4
        B5(6)=6*ALF5**5
        B5(7)=ALF5**6
        WRAD=FREQ*2*PI
        P1=(1.57-TETA)/WRAD
        P2=1/FREQ
        ALFN=(ALF1+ALF2+ALF3+ALF4+ALF5+ALFO)/6
        ALJ=1/ALFO
        WIDTH=2*DELT
*
*
STORAG IC1(2),IC2(3),IC3(4),IC4(5),IC5(6),...
```

```
*
*
        B1(3),B2(4),B3(5),B4(6),B5(7),...
        A1(1),A2(1),A3(1),A4(1),A5(1)
*
*
TABLE   IC1(1-2)=2*0.0,IC2(1-3)=3*0.0,IC3(1-4)=4*0.0,...
        IC4(1-5)=5*0.0,IC5(1-6)=6*0.0
*
*
DYNAMIC
        INPT=IMPULS(P1,P2)
        DIN=PULSE(INPT,WIDTH)
*
*
DERIVATIVE
*
*
        YIN=SINE(0.0,WRAD,TETA)
NOSORT
        DOUTO=REALPL(0.,ALJ,DIN)
        YOUTO=REALPL(0.,ALJ,YIN)
        DOUT1=TRNFR(1,2,IC1,A1,B1,DIN)
        YOUT1=TRNFR(1,2,IC1,A1,B1,YIN)
        DOUT2=TRNFR(2,3,IC2,A2,B2,DIN)
        YOUT2=TRNFR(2,3,IC2,A2,B2,YIN)
        DOUT3=TRNFR(3,4,IC3,A3,B3,DIN)
        YOUT3=TRNFR(3,4,IC3,A3,B3,YIN)
        DOUT4=TRNFR(4,5,IC4,A4,B4,DIN)
        YOUT4=TRNFR(4,5,IC4,A4,B4,YIN)
        DOUT5=TRNFR(5,6,IC5,A5,B5,DIN)
        YOUT5=TRNFR(5,6,IC5,A5,B5,YIN)
*
*       FOR MULTIPLIERS
*
        MULO=YOUTO*YIN
        MUL1=YOUT1*YIN
        MUL2=YOUT2*YIN
        MUL3=YOUT3*YIN
        MUL4=YOUT4*YIN
        MUL5=YOUT5*YIN
*
*       FOR RESPONSE OF LP
*
        RESO=REALPL(0.0,LPT,MULO)
        RES1=REALPL(0.0,LPT,MUL1)
        RES2=REALPL(0.0,LPT,MUL2)
        RES3=REALPL(0.0,LPT,MUL3)
        RES4=REALPL(0.0,LPT,MUL4)
        RES5=REALPL(0.0,LPT,MUL5)
*
*       FOR SECOND SET OF MULTIPLIERS
*
        MULDO=DOUTO*RESO
        MULD1=DOUT1*RES1
        MULD2=DOUT2*RES2
        MULD3=DOUT3*RES3
        MULD4=DOUT4*RES4
        MULD5=DOUT5*RES5
*
*       FOR ADDER
*
        XCOR=MULDO+MULD1+MULD2+MULD3+MULD4+MULD5
        ARGU=ALFN*TIME
        EXPO=EXP(ARGU)/ALFN
        CORR = XCOR*EXPO
*
*
SAMPLE
PRINT   10E-05,MULDO,MULD1,MULD2,MULD3,MULD4,MULD5,YIN,...
        XCOR,CORR,EXPO
PREPAR  10E-05,YIN,XCOR,CORR,EXPO
```

For Transfer see
6-17

```
CONTRL  FINTIM=20E-03,DELT=20E-06,DELS=10E-05
GRAPH   TIME,XCOR
LABEL   XCOR VS TIME.
GRAPH   TIME,CORR
LABEL   CORR VS TIME.
GRAPH   TIME,EXPO
LABEL   EXPO VS TIME.
PRPLOT  ONLY
*
*
        CALL DRWG(1,1,TIME,YIN)
        CALL DRWG(2,1,TIME,XCOR)
        CALL DRWG(3,1,TIME,CORR)
        CALL DRWG(4,1,TIME,EXPO)
*
*
TERMINAL
        CALL ENDRW(NPLOT)
END
STOP
//PLOT.STEPLIB  DD  DSN=SYS3.DSLPLOT,UNIT=2321,...
  VOL=SER=CEL009,DISP=SHR
//PLOT.SYSIN DD *
```

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